CA - Foundation

Paper - 3 (100 marks)

Maths (8 chapters) (40)	Statistics (6 chapters) (40)	Logical reasoning (4 chapters) (20)
① Ratio, proportions, Logs, Indices	① Statistical Descripti. of data	① Number series, coding, Decoding and odd man out
② Time value of money OR (SI & CI)	② Measures of central tendency & Measures of dispersion	② Direction Tests
③ Sequences & series (AP & GP)	③ correlation & regression analysis	③ Blood Relations
④ Equations	④ probability & expected value	④ seating arrangements
⑤ Inequalities	⑤ probability Distri. OR Theoretical Distributions	
⑥ permutations & combinations	⑥ Index Numbers	
⑦ Sets, Functions & Relations		
⑧ Derivaties & Integration		

Ratios, proportions, Logs, Indices

- Ratio is a fraction used for comparison of 2 or more quantities which are
  - of same type/kind
  - expressed in same unit of measurement

- For example

  If Ram's weight is 55 kgs & shyam's weight is 60 kgs then we can say that

  Ratio of Ram, shyam's weight is $55:60 = 11:12$

  OR Ratio of shyam, Ram's weight is $60:55 = 12:11$

  $\therefore$ $11:12$ is the **simplest form** of $55:60$

- Ratio is generally expressed in **simplest form**

- $a:b$ can also be written $\frac{a}{b} = \frac{am}{bm}$ where $m \neq 0$

  $= \frac{(a/m)}{(b/m)}$

- All the terms of the ratio can be multiplied or divided by same **non-zero** number without changing the meaning of that ratio.

  $a:b = am:bm = \frac{a}{k} : \frac{b}{k} = ax:bx = \frac{ay}{by} = \frac{a/10}{b/10}$

  where $m, k, x, y \neq 0$

- In the Ratio $a:b$

  a = First term = Antecedent

  b = second term = consequent

In the ratio $a:b$ If	then Ratio is called as	examples
$a > b$ i.e. Antecedent > consequent	Ratio of greater inequality	8:7, 9:2, 11:3, 100:21
$a < b$ i.e. Antecedent < consequent	Ratio of Lesser inequality	11:19, 23:28, 55:91, 93:101
$a = b$ i.e. Antecedent = consequent	Ratio of equality	5:5, 10:10, 1:1, 90:90

simplest form of Ratio of equality is always 1:1

- Find simplest form of ratio of quantities

  (3 hrs 5 mins 8 secs), (8 hrs 10 mins 40 secs)

$\Rightarrow$

$$= \frac{(3\times60\times60)+(5\times60)+8}{(8\times60\times60)+(10\times60)+40}$$

$$= \frac{10,800+300+8}{28,800+600+40} = \frac{11108}{29440} = \frac{2777}{7360}$$

$$= 2777:7360$$

- Ratio can be expressed without unit of measurement (i.e. Ratios are unit-free)

Quantities	Ratio in simplest form
5 hrs, 5 mins	(5×60) mins : 5 mins $300 : 5 = 60 : 1$
5 hrs 10 mins, 10 hrs 15 mins	$\frac{(5 \times 60) + 10}{(10 \times 60) + 15} = \frac{310 \text{ mins}}{615 \text{ mins}}$ $= 62 : 123$
8 feets 6 inches, 12 feets 8 inches	Pls. remember 1 foot = 12 inches $1' = 12''$ $\frac{(8 \times 12) + 6}{(12 \times 12) + 8} = \frac{102 \text{ inches}}{152 \text{ inches}}$ $= 51 : 76$
8 meters 10 cms, 10 meters 18 cms	Pls remember 1 meter = 100 cms 1 cm = 10 mms 1 meter = 1000 mms $\frac{(8 \times 100) + 10}{(10 \times 100) + 18} = \frac{810 \text{ cms}}{1018 \text{ cms}}$ $= 810 : 1018 = 405 : 509$
5 mins 33 secs, 8 mins 45 secs	$\frac{(5 \times 60) + 33}{(8 \times 60) + 45} = \frac{333 \text{ secs}}{525 \text{ secs}}$ $= 333 : 525 = 111 : 175$
8 GB, 512 MB	Please remember 1024 bytes = 1 kB 1024 kB = 1 MB 1024 MB = 1 GB 1024 GB = 1 TB $\frac{(8 \times 1024) \text{ MB}}{512 \text{ MB}} = \frac{8192 \text{ MB}}{512 \text{ MB}}$ $= 16 : 1$

8.5 kgs, 880 gms	Please remember : 1 kg = 1000 gms $\frac{8500 \text{ gms}}{880 \text{ gms}} = \frac{850}{88} = \frac{425}{44}$ $= 425:44$
8.8 kms, 44000 mms	$\frac{(8.8 \times 1000 \times 100 \times 10) \text{ mms}}{44000 \text{ mms}}$ $= \frac{8800000}{44000} = \frac{200}{1} = 200:1$

- order of terms in a ratio is important

$$a:b \neq b:a$$

- If we interchange the position of antecedent & consequent, we get **Inverse Ratio**
- Inverse ratio of 5:7 is 7:5
  & pls remember $5:7 \neq 7:5$
- Inverse Ratio of Inverse ratio of a:b is a:b

- Inverse Ratio of Ratio of Greater inequality is Ratio of lesser inequality and vice versa.
- Inverse ratio of ratio of equality is Ratio of equality only.
- **Duplicate Ratio:** Ratio of squares of all the terms is known as duplicate ratio.

  Duplicate ratio of $a:b$ is $\frac{a \times a}{b \times b} = \frac{a^2}{b^2}$

  Duplicate Ratio of $7:9$ is $49:81$
- **Triplicate Ratio**: Ratio of cubes of all the terms is known as Triplicate ratio
- **sub-duplicate Ratio**: Ratio of square roots of all the terms is known as sub-duplicate ratio.
- **sub-Triplicate Ratio**: Ratio of cube-roots of all the terms is known as sub-triplicate ratio.

	I + S				
A Ratio	Duplicate Ratio	Triplicate Ratio	sub-dupli Ratio	sub-tripli. Ratio	Inverse Ratio
$p:q$	$p^2 : q^2$	$p^3 : q^3$	$\sqrt{p} : \sqrt{q}$	$\sqrt[3]{p} : \sqrt[3]{q}$	$q:p$
$3:11$	$9:121$	$27:1331$	$\sqrt{3} : \sqrt{11}$	$\sqrt[3]{3} : \sqrt[3]{11}$	$11:3$
$a^2 : b^3$	$(a^2)^2 : (b^3)^2$ $= a^4 : b^6$	$(a^2)^3 : (b^3)^3$ $= a^6 : b^9$	$\sqrt{a^2} : \sqrt{b^3}$ $= (a^2)^{1/2} : (b^3)^{1/2}$ $= a : b^{3/2}$	$\sqrt[3]{a^2} : \sqrt[3]{b^3}$ $= (a^2)^{1/3} : (b^3)^{1/3}$ $= a^{2/3} : b$	$b^3 : a^2$
$a^7 : p^{10}$	$a^{14} : p^{20}$	$a^{21} : p^{30}$	$(a^7)^{1/2} : (p^{10})^{1/2}$ $= a^{7/2} : p^5$	$a^{7/3} : p^{10/3}$	$p^{10} : a^7$
$3^{1/3} : 8^{1/7}$	$(3^{1/3})^2 : (8^{1/7})^2$ $= 3^{2/3} : 8^{2/7}$	$(3^{1/3})^3 : (8^{1/7})^3$ $= 3 : 8^{3/7}$	$(3^{1/3})^{1/2} : (8^{1/7})^{1/2}$ $= 3^{1/6} : 8^{1/14}$	$(3^{1/3})^{1/3} : (8^{1/7})^{1/3}$ $= 3^{1/9} : 8^{1/21}$	$8^{1/7} : 3^{1/3}$

Compounded Ratio or Ratio compounded

- compounded ratio of $2:3,\ x:y,\ m:n,\ p:q,\ e:f$ is $(2xmpe):(3ynqf)$

$$\text{compounded Ratio} = \left(\begin{matrix}\text{product of all}\\ \text{antecedents}\end{matrix}\right) : \left(\begin{matrix}\text{product of all}\\ \text{consequents}\end{matrix}\right)$$

- Find Ratio compounded of $3:7,\ 8:3,\ 5:11,\ 22:5,\ 14:17$

$\Longrightarrow$

$$\text{compounded ratio} = \frac{3 \times 8 \times 5 \times 22 \times 14}{7 \times 3 \times 11 \times 5 \times 17}$$

$$= \frac{32}{17} = 32:17$$

- Find compounded ratio of $8:13,\ 39:5,\ 25:18,\ 27:15,\ 30:8,\ 16:5,\ 50:2$

$\Longrightarrow$ compounded Ratio

$$= \left(\frac{\text{product of all antecedents}}{\text{product of all consequents}}\right)$$

$$= \frac{8 \times 39 \times 25 \times 27 \times 30 \times 16 \times 50}{13 \times 5 \times 18 \times 15 \times 8 \times 5 \times 2}$$

$$= \frac{3 \times 3 \times 16 \times 25}{1} = \frac{3600}{1} = 3600:1$$

- Find compounded ratio 5:7, 14:3, 9:2, 6:15, dupli. ratio of 5:2, Tripli. ratio of 4:5, sub-dupli. ratio of 25:49, subtripli. ratio of 343:64, inverse ratio of 2:9, Dupli. ratio of 3:2

$\Longrightarrow$

5:7, 14:3, 9:2, 6:15, 25:4, 64:125, 5:7, 7:4, 9:2, 9:4

$$\text{compounded Ratio} = \frac{5 \times 14 \times 9 \times 6 \times 25 \times 64 \times 5 \times 7 \times 9 \times 9}{7 \times 3 \times 2 \times 15 \times 4 \times 125 \times 7 \times 4 \times 2 \times 4}$$

$$= \frac{9 \times 3 \times 9}{1} = 243:1$$

- Find Duplicate ratio of Triplicate ratio of Inverse ratio of 5:8

$\Longrightarrow$ = Dupli ratio of Triplicate ratio of 8:5

= Dupli. ratio of $(8^3 : 5^3)$

= $(8^3)^2 : (5^3)^2$

= $8^6 : 5^6$

= $(2,62,144 : 15625)$

- Find Triplicate ratio of sub-duplicate ratio of Inverse ratio of 64:49

$\Longrightarrow$ Tripli. ratio of sub-dupli. ratio of 49:64

= Tripli ratio of 7:8

= $7^3 : 8^3$

= 343:512

. Find Duplicate ratio of sub-triplicate ratio of 8:11

$\Rightarrow$ Dupli. ratio of $\sqrt[3]{8} : \sqrt[3]{11}$

$= \left(\sqrt[3]{8}\right)^2 : \left(\sqrt[3]{11}\right)^2$

$= (2)^2 : \left(11^{1/3}\right)^2$

$= \left(4 : 11^{2/3}\right)$

Please remember

$\sqrt[m]{a} = a^{1/m}$

. Find Duplicate ratio of sub-duplicate ratio of 8:9

$\Rightarrow$ = Duplicate ratio of $\sqrt{8} : \sqrt{9}$

= 8:9

please remember

1) Duplicate ratio of sub-duplicate ratio of 5:7 is 5:7
2) Sub-duplicate ratio of Duplicate ratio of $x:y$ is $x:y$
3) Triplicate ratio of sub-triplicate ratio of $m:n$ is $m:n$
4) Sub-triplicate ratio of Triplicate ratio of $e:f$ is $e:f$
5) Inverse Ratio of Inverse ratio of 8:11 is 8:11

. Find compounded ratio of a:b & a:b

$\Rightarrow$ $a^2 : b^2$

1) If a ratio is compounded with itself we get its Duplicate ratio
2) If a ratio is compounded with its duplicate ratio then we get its triplicate ratio

. Find Ratio compounded of 5:8 and 8:5

$\Rightarrow$ $= (5 \times 8) : (8 \times 5)$

$= 40 : 40$

$= 1 : 1$

3) If a ratio is compounded with its inverse ratio then result is : Ratio of equality (i.e. 1:1)

Few calculator short-cuts

① $\sqrt{64} = 8$

$\sqrt[4]{512} = (512)^{1/4} = (512^{1/2})^{1/2} = 4.7568$

$\sqrt[8]{1000} = \left[(1000^{1/2})^{1/2}\right]^{1/2} = 2.3713$

$\sqrt[16]{25,000}$ = Take 25000 on calculator & press $\sqrt{}$ 4 times

$= 1.8830$

If I take a number on calculator & press $\sqrt{}$ button

	I get
once	2nd root
Twice	4th root
Thrice	8th root
Four times	16th root
Five time	32nd root

$\sqrt[32]{85,90,624} = 1.6469$ $\qquad$ $\sqrt[8]{263.9584} = 2.0076$

$\sqrt[16]{95800} = 2.0480$ $\qquad$ $\sqrt[64]{28,30,925.82} = 1.2612$

② $(1.1083)^{12} = 3.4346$

Take 1.1083 on calculator then press 'x'
then press =, =, =, = ----- till step count comes 13

$(1.2083)^{28} = 199.9372$

Take 1.2083 on calculator then press 'x'
then press =, =, = ----- till step count comes 29

$(1.1369)^{58} = 1705.6560$

$(1.1293)^{85} = 30817.1797$

$(2.8056)^{13} = 667320.9929$

③

$\sqrt[5]{85} = 85^{1/5}$

$= 85^{0.20}$

$= (2.432\,490\,8634)$

Cross-check

$(2.432\,490\,8634)^{5} =$

How to find a^b on calculator ?

⇒ Enter 'a'
$\sqrt{}$ 12 times
Deduct 1
Multiply by 'b'
Add 1
(x=) 12 times

↓
You will get answer on calculator screen

$(58500)^{0.8196} = 8092.91245$

$\sqrt[7]{50,85,000} = (5085000)^{1/7} = 9.11152$

$\sqrt[18]{6456983256} = (6456983256)^{1/18} = 3.51903$

① If $a:b = 2:3$

$b:c = 3:5$

Find $a:b:c$ and $a:c$ and $c:a$

$\Longrightarrow$ $a:b:c = 2:3:5$

$a:c = 2:5$

$c:a = 5:2$

② If $a:b = 5:6$

$b:c = 8:13$ Find $a:b:c$, $a:c$, $c:a$

$\Longrightarrow$

$a:b = 5:6 = 20:24$

$b:c = 8:13 = 24:39$

$\therefore$ $a:b:c = 20:24:39$

$a:c = 20:39$

$c:a = 39:20$

③ If $a:b = 5:9$, $b:c = 8:17$ Find $a:b:c$, $a:c$, $c:a$

$\Longrightarrow$ $a:b = 5:9 = 40:72$

$b:c = 8:17 = 72:153$

$\therefore$ $a:b:c = 40:72:153$

& $a:c = 40:153$

$c:a = 153:40$

④ If $a:b = 3:8$, $b:c = 5:11$, $c:d = 12:15$

Find $a:b:c:d$, $a:d$, $b:d$, $c:d$

$\Longrightarrow$ $a:b = 3:8 = 15:40$

$b:c = 5:11 = 40:88$

$\therefore$ $a:b:c = 15:40:88$

$c:d = 4:5 = 88:110$

$\therefore$ $a:b:c:d = 15:40:88:110$

$a:d = 15:110 = 3:22$

$b:d = 40:110 = 4:11$

$c:d = 88:110 = 8:10 = 4:5$

(5) If $a:b = 3:7$, $b:c = 2:5$, $c:d = 10:11$

Find $a:b:c:d$, $b:d$, $a:c$, $c:d$, $a:d$

$\Rightarrow$ $a:b = 3:7 = 6:14$

$b:c = 2:5 = 14:35$

$\therefore$ $a:b:c = 6:14:35 = 12:28:70$

$c:d = 10:11 = 70:77$

$\therefore$ $a:b:c:d = 12:28:70:77$

$b:d = 28:77 = 4:11$

$a:c = 12:70 = 6:35$

$c:d = 70:77 = 10:11$

$a:d = 12:77$

(6) If $a:b = 2:5$

$b:c = 6:9$

$c:d = 4:7$ Find $a:b:c:d$

$\Rightarrow$ $a:b = 2:5 = 12:30$

$b:c = 6:9 = 30:45$

$\therefore$ $a:b:c = 12:30:45 = 48:120:180$

$c:d = 4:7 = 180:315$

$\therefore$ $a:b:c:d = 48:120:180:315$

$= 16:40:60:105$

(7) $a:b = 2:7$

$a:c = 3:8$ Find $a:b:c:d:e$

$a:d = 5:11$

$a:e = 6:13$

$\Rightarrow$ $a:b = 2:7 = 30:105$

$a:c = 3:8 = 30:80$

$a:d = 5:11 = 30:66$

$a:e = 6:13 = 30:65$

$a:b:c:d:e = 30:105:80:66:65$

(8) $a:b = 5:2$

$a:c = 6:7$

$b:d = 15:8$

Find $a:b:c:d$

$\Longrightarrow$ $a:b = 5:2 = 30:12$

$a:c = 6:7 = 30:35$

$\therefore$ $a:b:c = 30:12:35 = 150:60:175$

$b:d = 15:8 = 60:32$

$\therefore$ $a:b:c:d = 150:60:175:32$

. $\sqrt[10]{55000} = 55{,}000^{1/10} = 2.9826$

. $\sqrt[16]{8582968256 00} = 5.56996$

. $\sqrt[20]{258629815361} = (258629815361)^{1/20}$

$= 3.735736$

. $(1.125896)^{80} = 13178.382127 5$

. Find Triplicate ratio of sub-duplicate ratio of sub-duplicate ratio of Inverse ratio of $\sqrt{3} : \sqrt[5]{11}$

$\Longrightarrow$ Tripli, subdupli, sub-dupli of $(11^{1/5}) : (3)^{1/2}$

$= $ Tripli. ratio of $\left[(11^{1/5})^{1/2}\right]^{1/2} : \left[(3^{1/2})^{1/2}\right]^{1/2}$

$= $ Triplicate ratio of $11^{1/20} : 3^{1/8}$

$= (11^{1/20})^3 : (3^{1/8})^3$

$= 11^{3/20} : 3^{3/8}$

. Ratio of 3 or more terms is known as

continued ratio

example : 3:5:8:11

Ratio	Its simplest form
1.50 : 2.50	15:25 = 3:5
3 : 5	3:5
8 : 88	1:11
13 : 39	1:3
$\sqrt{5} : \sqrt{7}$	$\sqrt{5} : \sqrt{7}$
96 : 33	32:11
$\frac{1}{3} : \frac{5}{3}$	1:5
$\frac{2}{7} : \frac{8}{11}$	22 : 56 = 11:28
1.50 : 2.75	150 :275 = 6:11
$2\frac{1}{3} : 5\frac{1}{3}$	$\frac{7}{3} : \frac{16}{3}$ = 7:16
$\sqrt{25} : \sqrt{81}$	5:9
1.22 : 6.56	122 : 656 = 61:328
$8\frac{1}{7} : 13\frac{2}{7}$	$\frac{57}{7} : \frac{93}{7}$ = 57:93 = 19:31
$8\frac{2}{9} : 15\frac{3}{7}$	$\frac{74}{9} : \frac{108}{7}$ = 518 : 972 = 259:486
81 : 2.43	8100 : 243 = 900 : 27 = 100:3

- If $a:b = 5:8$

  $c:d = 3:4$

  $a:d = 7:1$ Find $a:b:c:d$

$\Longrightarrow$ $a:b = 5:8 = 35:56$

$a:d = 7:1 = 35:5$

$a:b:d = 35:56:5 = 140:224:20$

$c:d = 3:4 = 15:20$

$\therefore$ $a:b:c:d = 140:224:15:20$

- If $a:c = 13:5$

  $b:d = 14:9$

  $a:d = 2:15$

  Find $a:b:c:d$

$\Longrightarrow$ $a:c = 13:5 = 26:10$

$a:d = 2:15 = 26:195$

$a:c:d = 26:10:195 = 234:90:1755$

$b:d = 14:9 = 2730:1755$

$a:b:c:d = 234:2730:90:1755$

$= 78:910:30:585$

- If $a:d = 9:8$ $b:c = 6:5$

  $a:c = 12:1$ Find $a:b:c:d$

$\Longrightarrow$ $a:d = 9:8 = 108:96$

$a:c = 12:1 = 108:9$

$a:c:d = 108:9:96 = 540:45:480$

$b:c = 6:5 = 54:45$

$a:b:c:d = 540:54:45:480$

$= 180:18:15:160$

• Multiplying ratio

Mr. A has Income of ₹ 1,00,000 in Year 2023.
In 2024 his income changes in the ratio of 4:7.
Find his New Income?

old	New
4	7
1,00,000	?

$\Longrightarrow$

$? = 1,00,000 \times \frac{7}{4} = 1,75,000$

New Income = old Income × Multiplying ratio
= old Income × (Inverse ratio of given ratio)

New quantity = old quantity × Multiplying ratio

old quantity = New quantity × Given ratio

where Multiplying ratio = (Inverse ratio of given ratio)

• India's population in a year is 140 crores. Next census shows that population has changed in the ratio of 7:9. Find New population?

$\Longrightarrow$ New population = old population × Multiplying ratio

$= 140 \text{ crores} \times \frac{9}{7}$

$= 180 \text{ crores}$

Also pls. remember

New population × Given ratio = old population

- Ratio of 2 quantities is 3:4. on adding 6 in both quantities it becomes 4:5. Find those quantities.

$\Longrightarrow$ suppose those quantities are $3x$ & $4x$

$$\frac{3x+6}{4x+6} = \frac{4}{5}$$

$$15x + 30 = 16x + 24$$

$$6 = x$$

$$\therefore \quad x = 6$$

$\therefore$ Those 2 quantities are $3x = 3 \times 6 = 18$

$4x = 4 \times 6 = 24$

- Ratio of 2 quantities of 4:5 on adding 6 in first term and 45 in second term ratio becomes 1:2. Find those 2 quantities.

$\Longrightarrow$ 2 quantities are $4x$ and $5x$

$$\frac{4x+6}{5x+45} = \frac{1}{2}$$

$$8x + 12 = 5x + 45$$

$$3x = 33$$

$$x = 11$$

$\therefore$ Those 2 quantities are $4x = 4 \times 11 = 44$

$5x = 5 \times 11 = 55$

- Ratio of Father and son's age is 16:1 after 8 years Ratio of their age becomes 4:1. Find present age of Father and son.

$\Longrightarrow$ Let present age of Father, son be $16x$, x

$$\frac{16x+8}{x+8} = \frac{4}{1} \qquad \therefore \quad 16x + 8 = 4x + 32$$

$$12x = 24$$

$$\boxed{x = 2}$$

$\therefore$ present age of Father, son is 32, 2 years resp.

- Anand's Income in 2024 in ₹ 20 crores. In comparison with year 2023 his Income has changed in the ratio of 47:50. Find Anand's Income in year 2023

$\Rightarrow$ Old Income = New Income × Given ratio

$= 20 \text{ crores} \times \left(\frac{47}{50}\right)$

$= 18.80 \text{ crores}$

old income	New income
47	50
?	20 crores

Please remember

New Income = old Income × multiplying ratio

old Income = New Income × Given ratio

where $\left(\begin{matrix}\text{multiplying} \\ \text{ratio}\end{matrix}\right) = \left(\begin{matrix}\text{Inverse ratio of} \\ \text{given ratio}\end{matrix}\right)$

- Ratio of Ram and Shyam's age is 3:4. 10 years before their age was in the ratio of 1:2. Find sum of present ages of Ram & Shyam?

$\Rightarrow$ Let present age of Ram, Shyam be $3x$ & $4x$

$$\frac{3x-10}{4x-10} = \frac{1}{2}$$

$$6x - 20 = 4x - 10$$

$$2x = 10$$

$$x = 5$$

$\therefore$ present ages are 15, 20

sum of their present age $= 15+20$

$= 35$ years

- Find Duplicate ratio of 2:5

(a) 8:125 (b) 8:50 (c) $\sqrt{2}:\sqrt{5}$ (d) None of these

- Find sub-duplicate ratio of 1250:25

(a) $\sqrt{1250}:25$ (b) $\sqrt{50}:5$ (c) $1250^2:25^2$ (d) None of these

Answer : $\sqrt{\frac{1250}{25}} = \sqrt{\frac{50}{1}} = \sqrt{50}:1$

- A mixture contains water, milk is the ratio of 5:1 on adding 10 litres of water the ratio becomes 7:1. Find milk qty in original mixture ?

$\Longrightarrow$ Let qty of water $= 5x$, milk $= x$ } in original mixture

$$\frac{5x+10}{x} = \frac{7}{1}$$

$$5x+10 = 7x$$

$$10 = 2x$$

$$\therefore x = 5$$

milk qty in original mixture = 5 litres

& water qty in original mixture = 25 litres

- There are 2 containers, Ratio of milk, water in first, second container is 5:2 and 9:11. First contain has 14 litres & second container has 35 litres. If these mixtures are put in third container Find milk qty : water qty in 3rd container.

	milk	water	Total
1st container	$\frac{5}{7} \times 14 = 10$	$\frac{2}{7} \times 14 = 4$	14
2nd container	$\frac{9}{20} \times 35 = 15.75$	$\frac{11}{20} \times 35 = 19.25$	35
Total	25.75	23.25	

Answer : $25.75 : 23.25$

$= 2575 : 2325$

$= 103 : 93$

- Distribute ₹5,00,000 among A, B, C in the ratio of 7:3:20

$\Rightarrow$ A's share $= 7/30 \times 5,00,000 = 1,16,666.666666$

B's share $= 3/30 \times 5,00,000 = 50,000$

C's share $= 20/30 \times 5,00,000 = 3,33,333.333333$

(OR)

$7x + 3x + 20x = 5,00,000$

$30x = 5,00,000$

$x = 16,666.6666666$

$\therefore \quad 7x = 1,16,666.6666$

$3x = 50,000$

$20x = 3,33,333.333333$

- Distribute ₹600 among A, B, C, D in the ratio of $\left(2\frac{1}{3} : 5\frac{1}{3} : 8\frac{2}{3} : 10\frac{1}{3}\right)$

$\Rightarrow \quad 2\frac{1}{3} : 5\frac{1}{3} : 8\frac{2}{3} : 10\frac{1}{3}$

$= \frac{7}{3} : \frac{16}{3} : \frac{26}{3} : \frac{31}{3}$

$= 7 : 16 : 26 : 31$

Share of A $= \frac{7}{80} \times 600 = 52.50$ D: $\frac{31}{80} \times 600 = 232.50$

B $= \frac{16}{80} \times 600 = 120$

C $= \frac{26}{80} \times 600 = 195$

- Distribution ₹50,000 among A, B, C, D in the ratio of $\frac{1}{3} : \frac{1}{5} : \frac{2}{3} : \frac{2}{9}$

$\Rightarrow$ $\frac{1}{3} : \frac{1}{5} : \frac{2}{3} : \frac{2}{9}$

Lets multiply all terms by 45

$= 15 : 9 : 30 : 10$

A's share $= \frac{15}{64} \times 50{,}000 = 11718.75$

B's share $= \frac{9}{64} \times 50{,}000 = 7031.25$

C's share $= \frac{30}{64} \times 50{,}000 = 23437.50$

D's share $= \frac{10}{64} \times 50{,}000 = 7812.50$

Proportion

- When 4 quantities a, b, c, d are said to be in proportion ?

$\Rightarrow$ • when $a : b = c : d$ then a, b, c, d are said to be in proportion.

- When Ratio of 1st to 2nd term is equal to ratio of 3rd to 4th term then 4 terms are said to be in proportion.

When $P : q = r : s$

$\frac{P}{q} = \frac{r}{s}$

$ps = qr$

(product of extremes) = (product of means)

} P, q, r, s are said to be in proportion

- 35, 40, 7.50, 2k are in proportion. Find k

$\Rightarrow$ product of extremes = product of means

$(35 \times 2k) = (40 \times 7.50)$

$70k = 300$

$k = \frac{300}{70} = \frac{30}{7} = 4.2857$

4 quantities in proportion	value of k
a, 2b, 3m, 5k	$5ak = 6bm$ $\therefore k = \frac{6bm}{5a}$
15, 20, 25, 6k	$90k = 500$ $k = \frac{500}{90} = \frac{50}{9} = 5.5555$
35, −2k, 700, 830	$35 \times 830 = -1400k$ $\therefore k = \frac{35 \times 830}{-1400} = -20.75$
86, 43, 9k, 200	$86 \times 200 = 387k$ $k = 44.4444$
5.50, 7.50, 5k, 15	$5.50 \times 15 = 7.50 \times 5k$ $k = (82.50/37.50) = 2.20$
3k, 15, 21, 30	$3k \times 30 = 15 \times 21$ $90k = 315 \quad \therefore k = 3.50$
18, 28, $\frac{-3k}{8}$, 30	$18 \times 30 = 28 \times -3k/8$ $540 = -10.50k \quad k = -51.42857$

- When 4 quantities a, b, c, d are said to be in continued proportion ?

⟹ When $a:b = b:c = c:d$ then a, b, c, d are said to be in continued proportion.

- when $y:v = v:w = w:x$
  then y, v, w, x are said to be continued propⁿ

- 5, 20, 30, 120 these 4 quantities are in proportion but not in continued proportion

  as $5:20 = 30:120 \neq 20:30$

- 8, 12, 18, 27 as $8:12 = 12:18 = 18:27 = 2:3$

  These 4 quantities are in continued proportion

  ∴ They are in proportion also

- If 4 quantities are in continued propn then they must be in proportion

  but

  If 4 quantities are in proportion, It is not necessary that they are in continued proportion.

4 quantities	Whether quantities are in	
	proportion ?	continued proportion ?
$1, 2, 3, 4$	No	No
$50, 60, 25, 30$	Yes	No
$\frac{2}{3}, \frac{1}{3}, \frac{8}{7}, \frac{4}{7}$	Yes	No
$4, 6, 9, 13.50$	Yes	Yes
a, a^2, b, ab	Yes	No
x, x^2, x^3, x^4	Yes	Yes
$5, 20, 80, 320$	Yes	Yes
$5, 15, 155, 1555$	No	No
$20 : 200 : 2000 : 20000$	Yes	Yes
$5, 55, 555, 5555$	No	No
a, ax, b, bx	Yes	No
$8, 12, 18, 27$	Yes	Yes
$1.25, 2.50, 5, 7.50$	No	No
k, kx, x, kx	No	No
x^3, x^8, x^{13}, x^{18}	Yes	Yes

- $\frac{5}{7}, \frac{8}{11}, \frac{9}{13}, \frac{k}{18}$ are in proportion.

Find value of k.

$\Longrightarrow$ product of extremes = product of means

$$\frac{5}{7} \times \frac{k}{18} = \frac{8}{11} \times \frac{9}{13}$$

$$k = \frac{8}{11} \times \frac{9}{13} \times 18 \times \frac{7}{5}$$

$$k = \left(\frac{9072}{715}\right) = 12.6881$$

- a, b, c, d, e, f, g, h, i, j are said to be continued proportion when

$a:b = b:c = c:d = e:f = d:e = f:g = g:h = h:i = i:j$

When three terms p, q, r are said to be in proportion ?

$\Longrightarrow$

If $p:q = q:r$ i.e. $\frac{p}{q} = \frac{q}{r}$ i.e. $q^2 = pr$

then p, q, r are said to be in proportion

If p, q, r are in proportion then they are in continued propn also

- If a, b, c are in proportion then $b^2 = ac$

. Find 4th proportional to 8, 12, 20

$\Rightarrow$ Let 4th proportional be m

then 8, 12, 20, m are proportion

$8m = 12 \times 20$

$\boxed{m = 30}$ $\therefore$ 4th proportional to 8, 12, 20 is 30.

. Find 4th proportional to $\frac{3}{8}, \frac{7}{8}, \frac{9}{11}$

$\Rightarrow$ $\frac{3}{8}, \frac{7}{8}, \frac{9}{11}, m$ are propn

$$\frac{3m}{8} = \frac{7}{8} \times \frac{9}{11}$$

$$m = \frac{7}{\cancel{8}} \times \frac{\cancel{9}^{3}}{11} \times \frac{\cancel{8}}{\cancel{3}} = \frac{21}{11} = 1.909090$$

. Find 3rd proportional to 8, 12

$\Rightarrow$ 8, 12, m must be proportion.

$\therefore$ $12^2 = 8m$

$8m = 144$ $\therefore$ 8, 12, 18 are in propn

$\boxed{m = 18}$

. Find mean proportional to 10 and 22.50

$\Rightarrow$ 10, k, 22.50 are in proportion

$\therefore$ $k^2 = 10 \times 22.50$

$k^2 = 225$

$k = 15$

. Find mean proportional to 28 & 59

$\Rightarrow$ 28, x, 59 are in proportion

$\therefore$ $x^2 = 28 \times 59$

$x^2 = 1652$ $\therefore$ $x = \sqrt{1652}$

$x = 40.6448$

Properties of proportion

If a, b, c, d are in proportion

i.e. $\frac{a}{b} = \frac{c}{d}$ then

① $\frac{b}{a} = \frac{d}{c}$ ..... Invertendo

② If $\frac{a}{b} = \frac{c}{d}$

then $\frac{a}{c} = \frac{b}{d}$ .... Alternendo

$\frac{5}{8} = \frac{10}{16}$

then $\frac{5}{10} = \frac{8}{16}$

③ If $\frac{a}{b} = \frac{c}{d}$

then $\frac{a}{b} + 1 = \frac{c}{d} + 1$

$\left(\frac{a+b}{b}\right) = \left(\frac{c+d}{d}\right)$ ... componendo

$\frac{5}{8} = \frac{50}{80}$

then $\frac{5+8}{8} = \frac{50+80}{80}$

$\frac{13}{8} = \frac{130}{80}$

④ If $\frac{a}{b} = \frac{c}{d}$

then $\frac{a}{b} - 1 = \frac{c}{d} - 1$

$\frac{a-b}{b} = \frac{c-d}{d}$ ..... Dividendo

⑤ If $\frac{a}{b} = \frac{c}{d}$

$$\frac{a+b}{b} = \frac{c+d}{d} \quad \text{-----①}$$

$$\frac{a-b}{b} = \frac{c-d}{d} \quad \text{-----②}$$

then

Eqn ① ÷ Eqn ②

$$\frac{(a+b)/b}{(a-b)/b} = \frac{(c+d)/d}{(c-d)/d}$$

$$\left(\frac{a+b}{a-b}\right) = \left(\frac{c+d}{c-d}\right) \quad \text{------- (componendo \& Dividendo)}$$

If $\frac{10}{8} = \frac{5}{4}$

$\therefore \frac{10+8}{10-8} = \frac{5+4}{5-4}$

$\frac{18}{2} = \frac{9}{1}$

⑥ If $\frac{a}{b} = \frac{c}{d}$

then

$\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$ .... Addendo

$\frac{8}{14} = \frac{24}{42}$

$\frac{8}{14} = \frac{24}{42} = \frac{8+24}{14+42}$

$= \frac{32}{56}$

$\frac{5}{28} = \frac{25}{140} = \frac{5+25}{28+140}$

$\frac{5}{28} = \frac{25}{140} = \frac{30}{168}$

⑦ If $\frac{a}{b} = \frac{c}{d}$ then

$$\frac{a}{b} = \frac{c}{d} = \frac{a-c}{b-d}$$ ----- Subtrahendo

$$\frac{64}{112} = \frac{16}{28} = \frac{64-16}{112-28} = \frac{48}{84} = \frac{4}{7}$$

$$\frac{105}{24} = \frac{70}{16} = \frac{105-70}{24-16}$$

If p, q, r, s are in proportion

i.e. $\frac{p}{q} = \frac{r}{s}$ then

As per

① Invertendo $\Longrightarrow \frac{q}{p} = \frac{s}{r}$

② Componendo $\Longrightarrow \frac{p+q}{q} = \frac{r+s}{s}$

③ Dividendo $\Longrightarrow \frac{p-q}{q} = \frac{r-s}{s}$

④ Compo & Divi $\Longrightarrow \frac{p+q}{p-q} = \frac{r+s}{r-s}$

⑤ Alternendo $\Longrightarrow \frac{p}{r} = \frac{q}{s}$

⑥ Addendo $\Longrightarrow \frac{p}{q} = \frac{r}{s} = \frac{p+r}{q+s}$

⑦ Subtrahendo $\Longrightarrow \frac{p}{q} = \frac{r}{s} = \frac{p-r}{q-s} = \frac{r-p}{s-q}$

. If $\frac{a}{7} = \frac{b}{8}$ Find value of $\left(\frac{8a-9b}{4b}\right)$

$\Rightarrow$

$\frac{8a}{8\times7} = \frac{b\times9}{8\times9}$

$\frac{8a}{56} = \frac{9b}{72} = \frac{8a-9b}{56-72}$

$\frac{b}{8} = \frac{8a-9b}{-16}$

$\frac{8a-9b}{-16} = \frac{4b}{32}$

$\frac{8a-9b}{4b} = \frac{-16}{32}$

$= -1/2$

OR

$\frac{a}{7} = \frac{b}{8} = k$

$\therefore a = 7k,\ b = 8k$

$\frac{8a-9b}{4b}$

$= \frac{8(7k)-9(8k)}{4(8k)}$

$= \frac{56k-72k}{32k} = \frac{-16k}{32k}$

$= -1/2$

If $\frac{a}{b} = \frac{c}{d}$ then

$\frac{a}{b} = \frac{c}{d} = \left(\frac{a+c}{b+d}\right) = \left(\frac{a-c}{b-d}\right) = \left(\frac{c-a}{d-b}\right)$

.... Addendo & subtrahendo

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c-e}{b+d-f} = \frac{a-c+e}{b-d+f}$

- If $\frac{a}{10} = \frac{c}{12} = \frac{b}{19}$ Find value of $\left(\frac{12a+19b-8c}{7c}\right)$

$\Rightarrow$ $a = 10m,\ c = 12m,\ b = 19m$

$$\frac{12a+19b-8c}{7c} = \frac{120m + 361m - 96m}{84m}$$

$$= \frac{385m}{84m} = \frac{385}{84} = \frac{55}{12} = 4.5833333$$

- If $\frac{a}{b} = \frac{c}{d}$ then $\frac{(a+c)^3}{(b+d)^3} = \frac{a^3+c^3}{b^3+d^3}$

---- True / False

$\Rightarrow$

$$\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$$

$$\left(\frac{a}{b}\right)^3 = \left(\frac{c}{d}\right)^3 = \left(\frac{a+c}{b+d}\right)^3$$

$$\frac{a^3}{b^3} = \frac{c^3}{d^3} = \frac{(a+c)^3}{(b+d)^3} \quad \text{----- (1)}$$

$$\frac{a^3}{b^3} = \frac{c^3}{d^3} = \frac{a^3+c^3}{b^3+d^3} \quad \text{---- (2)}$$

$\therefore$ $\frac{(a+c)^3}{(b+d)^3} = \frac{a^3+c^3}{b^3+d^3}$ ----- Given statement is True

Some Important formulae from school

1. $(a+b)^2 = a^2+b^2+2ab$
2. $(a^2+b^2) = (a+b)^2-2ab = (a-b)^2+2ab$
3. $(a-b)^2 = a^2+b^2-2ab$
4. $(a-b)^3 = a^3-b^3-3ab(a-b) = a^3-b^3+3ab(b-a)$
5. $(a+b)^3 = a^3+b^3+3ab(a+b) = a^3+b^3+3a^2b+3ab^2$
6. $(a^3+b^3) = (a+b)^3-3ab(a+b)$
7. $(a^2-b^2) = (a+b)(a-b)$
8. $(a^3-b^3) = (a-b)(a^2+ab+b^2)$
9. $(a^3+b^3) = (a+b)(a^2-ab+b^2)$
10. $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$
11. $(a-b+c)^2 = a^2+b^2+c^2-2ab-2bc+2ac$
12. $(a-b-c)^2 = a^2+b^2+c^2-2ab+2bc-2ac$
13. $(a-b)^2 = (a+b)^2-4ab$
14. $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$

Logarithms (Log)

① $\text{Log } x = (\text{characteristic of } x + \text{Mantissa of } x)$

Integer part = characteristic

Demical part = Fractional part = mantissa

- Log x
  - characteristic of x
    - If $x \geq 1$ → [(No. of digits before decimal point) − 1]
    - If $0 < x < 1$ → [(No. of zeros immediately after decimal point) + 1] Bar
  - mantissa of x → From Log-Table

x	characteristic of x	Mantissa of x	Log x
785.9	2	0.8954	2.8954
23.89	1	0.3783	1.3783
85921.00	4	0.9341	4.9341
7.000	0	0.8451	0.8451
58.976	1	0.7707	1.7707
81936	4	0.9135	4.9135
285.83	2	0.4560	2.4560
6.3923	0	0.8056	0.8056
18.1937	1	0.2598	1.2598
8.000	0	0.9031	0.9031

x	characteristic of x	Mantissa of x	$\text{Log}\, x$
69.293	1	0.8407	1.8407
6.6689	0	0.8241	0.8241
13.2093	1	0.1209	1.1209
10.00	1	0.0000	1
100.0	2	0.0000	2
1000	3	0.0000	3
10,000	4	0.0000	4
1.000	0	0.0000	0
5,00,000	5	0.6990	5.6990
50,000	4	0.6990	4.6990
5,000	3	0.6990	3.6990
500	2	0.6990	2.6990
50	1	0.6990	1.6990
5	0	0.6990	$\overline{0}.6990$
0.50	-1	0.6990	$= \overline{1}.6990$ -0.3010
0.050	-2	0.6990	$= \overline{2}.6990$ -1.3010
0.005	-3	0.6990	$\overline{3}.6990$ -2.3010
0.0005	-4	0.6990	$= \overline{4}.6990$ -3.3010
0.00005	-5	0.6990	$\overline{5}.6990$ $= -4.3010$

$\text{Log}\ 123.8 = 2.0927$

$\text{Log}\ 89.89 = 1.9537$

$\text{Log}\ 8.289 = 0.9185$

$\text{Log}\ 11.03 = 1.0425$

$\text{Log}\ 0.000008989 = \overline{6}.9537 = -6 + 0.9537$

$= -5.0463$

$\log 0.0002283 = \bar{4}.3585$

$= -4 + 0.3585 = -3.6415$

① $\log_8 23$ = Log of 23 to the base 8

$= \frac{\log 23}{\log 8} = \frac{1.3617}{0.9031} = 1.5078$

② $\log_b a = \left(\frac{\log a}{\log b}\right)$

③ $\log_{248} 75$

$= \frac{\log 75}{\log 248}$

$= \frac{1.8751}{2.3945}$

$= 0.78308$

- $\log_{10} 58 = \frac{\log 58}{\log 10} = \frac{1.7634}{1.000} = 1.7634$

common base of Logs is '10'

Natural base of Logs is 'e'

where e = exponential factor

= 2.7183 (approx)

① i) $\log_{10} 58 = 1.7634$

$10^{1.7634} = 58$

ii) $\log 234 = 2.3692$

$10^{2.3692} = 234$

If $\log_b a = k$ then $b^k = a$

iii) $\log_{10} 100 = 2$, $\log_{10} 10 = 1$

$\log_{10} 1000 = 3$, $\log_{10} 1 = 0$

If $\log_z x = m$ then $z^m = x$

A.log 2.3369 = 217.2

A.log 1.3369 = 21.72

A.log 0.8263 = 6.704

A.log 3.8985 = 7916.00

A.log 0.0981 = 1.253

A.log 0.0036 = 1.008

A.log 2.5893 = 388.5

A.log 4.6333 = 42980.00

A.log 5.9812 = 957600

A.log 10.6311 = 42770000000

$\text{Log } 23 = 1.3617 \qquad \text{A.log } 1.3617 = 23.00$

$\text{Log } 100 = 2.000 \qquad \text{A.log } 2.0000 = 100.0$

$$\Downarrow$$

$$\text{A.log}(\text{Log } 23) = \text{A.log}(1.3617) = 23$$

$$\text{Log}(\text{A.log } 2.000) = \text{Log}(100) = 2$$

$\therefore$ **$\text{A.log}(\log m) = m$**

$\text{Log}(\text{A.log } k) = k = \text{A.log}(\log k)$

- $\text{Log } 5 = 0.6990$, $\text{Log } 2 = 0.3010$, $\text{Log } 10 = 1.000$

$$1.0000 = 0.6990 + 0.3010$$

$$\text{Log } 10 = \text{Log } 5 + \text{Log } 2$$

$$\text{Log}(5 \times 2) = \text{Log } 5 + \text{Log } 2$$

$\therefore$ $\text{Log}_m(ab) = \text{Log}_m a + \text{Log}_m b$

- $\text{Log}(abc) = \text{Log}[(ab) \times c] = \text{Log } ab + \text{Log } c$
  $= \text{Log } a + \text{Log } b + \text{Log } c$

- $\text{Log}(pqrs) = \text{Log } p + \text{Log } q + \text{Log } r + \text{Log } s$

$m = 8 \times 5$

Find value of 'm'

$\Rightarrow$ $m = 8 \times 5$

Taking Log on both sides

$\log m = \log(8 \times 5)$

$= \log 8 + \log 5$

$= 0.9031 + 0.6990$

$\log m = 1.6021$

Taking A.log on both sides

$A.\log(\log m) = A.\log 1.6021$

$\boxed{m = 40.00}$

$\log_2 8 = ?$

$\log_2 8 = m$

If $\log_b a = k$ then $b^k = a$

If $\log_2 8 = m$

then $2^m = 8$

$2^m = 2^3$

$m = 3$

$\therefore \log_2 8 = 3$

$\log_2 8$

$= \frac{\log 8}{\log 2}$

$= \frac{\log(2 \times 2 \times 2)}{\log 2}$

$= \frac{\log 2 + \log 2 + \log 2}{\log 2}$

$= \frac{3\log 2}{\log 2}$

$= 3$

$\log_2 8$

$= \frac{\log 8}{\log 2}$

$= \frac{0.9031}{0.3010}$

$= 3$

How to find Log x on calculator?

⟹ ① Enter x
② $\sqrt{}$ 15 times
③ Deduct 1
④ multiply by 14230.9635

You will get answer of Log x on calculator

Log 28 = 1.4472

Log 81.37	= 1.91059
Log 998.11	= 2.99949
Log 87.33	= 1.94129
Log 1,05,990	= 5.02615
Log 383.1	= 2.58354
Log 1.0086	= 0.0037188
Log 0.007581	= −2.120115
Log 0.00005325	= −4.273039

Log x = characteristic of x + Mantissa of x

A.log (log m) = m

Log (A.log k) = k = A.Log (Log k)

Log (a × b) = Log a + Log b

Log (abc) = Log a + Log b + Log c

$\log_b a = \frac{\log a}{\log b}$

common base of Log is 10
Natural base of Log is e

If $\log_b a = k$ then $b^k = a$

$\log_{10} 1000 = 3.00$

$\log_{10} 100 = 2.00$

$\log_{10} 10 = 1.00$

$\log_{10} 1.00 = 0$

$\log_{10} 234 = 2.3692$

$\therefore 10^{2.3692} = 234$

$\log(2^6)$

$= \log(2 \times 2 \times 2 \times 2 \times 2 \times 2)$

$= \log 2 + \log 2 + \log 2 + \log 2 + \log 2 + \log 2$

$\log(2)^6 = 6 \cdot \log 2$

$\therefore \log(a)^b = b \cdot \log a$

$\log 3^8 = 8 \log 3$

$\log 11^{-3.20} = -3.20(\log 11)$

$\log e^y = y \cdot \log e$

$\log_c a^b = b \cdot \log_c a$

$= b \times \frac{\log a}{\log c}$

$\frac{8}{32} = \frac{2^3}{2^5} = 2^{3-5} = 2^{-2} = \frac{1}{2^2}$

$\therefore a^{-m} = \frac{1}{a^m}$

OR $a^m = \frac{1}{a^{-m}}$

$\sqrt[m]{k}$

$= k^{1/m}$

$$\log\left(\frac{a}{b}\right) = \log\left(a \times \frac{1}{b}\right)$$

$$= \log a + \log \frac{1}{b}$$

$$= \log a + \log\left(\frac{1}{b}\right)$$

$$= \log a + \log b^{-1}$$

$$= \log a + (-1 \times \log b)$$

$$= \log a - \log b$$

$$\log\left(\frac{ab}{c}\right) = \log(ab) - \log c$$

$$= \log a + \log b - \log c$$

$$\log x^{-y} = -y \cdot \log x$$

Summary of Logs

① $\log x$ = characteristic of x + mantissa of x

Integer part ⇒ characteristic

Fractional part ⇒ Mantissa

② ~~$A \cdot \log(\log k) = k = \log(A \cdot \log k)$~~

③ $\log(a \times b) = \log a + \log b$

④ $\log_k(xyz) = \log_k x + \log_k y + \log_k z$

⑤ $\log_n m = \left(\log m / \log n\right)$

⑥ $\log a^b = b \cdot \log a$

⑦ $\log m^{-k} = -k \cdot \log m$

⑧ If $\log_m a = p$ then $m^p = a$

⑨ $\log\left(\frac{a}{b}\right) = \log a - \log b$

⑰ common base of logs is '10'

Natural base of logs is 'e'

⑩ $\log_x\left(\frac{m}{n}\right)$

$= \log_x m - \log_x n$

⑪ $\log_{10} 10 = 1.00$

$\log_{10} 100 = 2$

$\log_{10} 1000 = 3$

$\log_b 1 = 0$

⑫ $\log a^1 = 0$

⑬ $\log_a a = 1$

⑭ $\log_b a \cdot \log_a b = 1$

⑮ $\log_b a \cdot \log_c b = \log_c a$

⑯ $\log\left(\frac{ab}{c}\right) = \log a + \log b - \log c$

Log 33.81

By using Log-table

= (characteristic of 33.81) + (mantissa of 33.81)

= 1 + 0.5290

= 1.5290

By using calculater

Enter 33.81

√ 15 times

−1

× 14230.9635

= 1.5291

Log$x$ = ?
Enter x
√ 15 times
Deduct 1
Multiply by 14230.9635

A Log x on calculater
Enter 'x'
Divide by 14230.9635
Add 1
(x =) 15 times

a^b = ?
Enter a
√ 12 times
Deduct 1
Multiply by 'b'
Add 1
(x =) 12 times

① $\log_{64} 512 = ?$

$$= \frac{\log 512}{\log 64} = \frac{\log 2^9}{\log 2^6} = \frac{9 \cdot \log 2}{6 \log 2} = \frac{9}{6} = \frac{3}{2} = 1.50$$

② $\log_{\sqrt{27}} 81^3 = \frac{\log 81^3}{\log \sqrt{27}} = \frac{\log (3^4)^3}{\log (3^3)^{1/2}}$

$$= \frac{\log 3^{12}}{\log 3^{3/2}} = \frac{12 \log 3}{\frac{3}{2} \log 3}$$

$$= 12 \times \tfrac{2}{3} = 8$$

③ $\log_{\sqrt[3]{32}} 1024 = \frac{\log 1024}{\log \sqrt[3]{32}}$

$$= \frac{\log 2^{10}}{\log (2^5)^{1/3}} = \frac{\log 2^{10}}{\log 2^{5/3}}$$

$$= \frac{10 \cdot \log 2}{\frac{5}{3} \log 2} = 10 \times \tfrac{3}{5} = 6$$

④ $\log_{\sqrt[8]{27}} \sqrt[9]{243} = \frac{\log \sqrt[9]{243}}{\log \sqrt[8]{27}}$

$$= \frac{\log \sqrt[9]{3^5}}{\log \sqrt[8]{3^3}} = \frac{\log (3^5)^{1/9}}{\log (3^3)^{1/8}} = \frac{\log 3^{5/9}}{\log 3^{3/8}}$$

$$= \frac{5/9 \cdot \log 3}{3/8 \cdot \log 3} = \frac{5}{9} \times \frac{8}{3} = \frac{40}{27}$$

Basic rules of Indices

① $2^3 \times 2^5$

$= 2\times2\times2\times2\times2\times2\times2\times2$

$= 2^8 = 2^{(3+5)}$

$\therefore \quad 2^3 \times 2^5 = 2^{3+5}$

$\therefore \quad a^m \times a^n = a^{m+n}$

② $\dfrac{3^8}{3^4} = \dfrac{3\times3\times3\times3\times\cancel{3}\times\cancel{3}\times\cancel{3}\times\cancel{3}}{\cancel{3}\times\cancel{3}\times\cancel{3}\times\cancel{3}} = 3^4 = 3^{8-4}$

$\dfrac{a^m}{a^n} = a^{(m-n)}$

③ $(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3 = 2^{3+3+3+3} = 2^{12}$

$= 2^{3\times4}$

$\therefore \quad (a^m)^n = a^{mn}$

④ $\left[(a^m)^n\right]^q = (a)^{mnq}$ ⑤ $a^x \times a^y \times a^z = a^{x+y+z}$

⑥ $\sqrt[m]{a} = a^{1/m}$

⑦ $\dfrac{2^5}{2^8} = 2^{5-8} = 2^{-3} = \dfrac{1}{2^3}$

$a^{-m} = \dfrac{1}{a^m}$ OR $a^m = \dfrac{1}{a^{-m}}$ OR $a^m \times a^{-m} = 1$

(8) $\frac{32}{32} = \frac{2^5}{2^5} = 2^{5-5} = 2^0 = 1$

(9) If $a^x = a^y$ then $x = y$

(10) If $a^x = b^x$ then $a = b$

(11) $(a)^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$

$(8)^{5/7} = \sqrt[7]{8^5}$

(12) $a^1 = a$, $a^0 = 1$

(13) $(2 \times 3)^4 = 1296$

$2^4 \times 3^4 = 16 \times 81 = 1296$

$(a \times b)^m = a^m \times b^m$

(14) $\left(\frac{6}{2}\right)^3 = 27$

$\frac{6^3}{2^3} = \frac{216}{8} = 27$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(15) $\left(\frac{ab}{c}\right)^m$

$= \frac{(ab)^m}{c^m}$

$= \frac{a^m \times b^m}{c^m}$

Rules of Indices

① $a^m \times a^n = a^{m+n}$

② $a^x \times a^y \times a^z = a^{(x+y+z)}$

③ $\left(\frac{a^m}{a^n}\right) = (a)^{m-n}$, $\frac{b^m}{b^n} = (b)^{m-n}$

④ $(a \times b)^x = a^x \times b^x$, $(xyz)^k = x^k \cdot y^k \cdot z^k$

⑤ $\left(\frac{a}{b}\right)^m = \left(a^m / b^m\right)$, $\left(\frac{xy}{z}\right)^m = \frac{x^m \cdot y^m}{z^m}$

⑥ $(a^x)^y = a^{xy}$, $\left\{\left[(m)^p\right]^q\right\}^r = (m)^{pqr}$

⑦ $a^{1/m} = \sqrt[m]{a}$

⑧ $a^{-m} = \frac{1}{a^m}$ OR $a^x = \frac{1}{a^{-x}}$ OR $a^p \times a^{-p} = 1$

⑨ $a^1 = a$, $a^0 = 1$

⑩ $a^{x/y} = \sqrt[y]{a^x}$, $x^{m/n} = \sqrt[n]{x^m}$

⑪ $a^{-1/m} = \frac{1}{a^{1/m}} = \left(1 / \sqrt[m]{a}\right)$

⑫ If $a^x = a^y$ then $x = y$

⑬ If $a^x = b^x$ then $a = b$

⑭ $\left(\frac{a^b \times a^c}{a^d}\right) = a^{b+c-d}$

⑮ $\left(\frac{a^b \times a^c}{a^e \times a^f}\right) = a^{b+c-e-f}$

① $2^3 \times (16)^8 \times (\sqrt{2})^9 = 2^3 \times (2^4)^8 \times (2^{1/2})^9$

$= 2^3 \times 2^{32} \times 2^{4.50}$

$= 2^{3+32+4.50} = (2)^{39.50}$

$= (2)^{79/2}$

② $3^8 \times 3^{-12} \times (\sqrt{3})^{16} \times (\sqrt[3]{3})^{18}$

$= 3^8 \times 3^{-12} \times (3^{1/2})^{16} \times (3^{1/3})^{18}$

$= 3^8 \times 3^{-12} \times 3^8 \times 3^6$

$= (3)^{8-12+8+6} = (3)^{10} = 59049$

③ $x^{a-b} \cdot x^{b-c} \cdot x^{c-a}$

$= x^{a-b+b-c+c-a} = x^0 = 1$

④ $(x^{a-b})^{a^2+ab+b^2} \cdot (x^{b-c})^{b^2+bc+c^2} \cdot (x^{c-a})^{c^2+ac+a^2}$

$= (x)^{(a-b)(a^2+ab+b^2)} \cdot (x)^{(b-c)(b^2+bc+c^2)} \cdot x^{(c-a)(c^2+ac+a^2)}$

$= (x)^{a^3-b^3} \cdot (x)^{b^3-c^3} \cdot (x)^{c^3-a^3} = (x)^{a^3-b^3+b^3-c^3+c^3-a^3}$

$= x^0 = 1$

⑤ $\left[\dfrac{(\sqrt{2})^9 \times 16^{-5/2} \times 64^{-7/2}}{32^{11/2}}\right] = \dfrac{(2^{1/2})^9 \times (2^4)^{-5/2} \times (2^6)^{-7/2}}{(2^5)^{11/2}}$

$= \dfrac{2^{9/2} \times 2^{-20/2} \times 2^{-42/2}}{2^{55/2}} = (2)^{\frac{9}{2}-\frac{20}{2}-\frac{42}{2}-\frac{55}{2}}$

$= (2)^{-54} = \left(\frac{1}{2}\right)^{54}$

⑥ $\sqrt[9]{\left[\sqrt[3]{4}\right]^{1/8}} = \left[\left[4^{1/3}\right]^{1/8}\right]^{1/9}$

$= (4)^{\frac{1}{3}\times\frac{1}{8}\times\frac{1}{9}} = (2^2)^{\frac{1}{216}} = \left(2^{\frac{2}{216}}\right) = \left(2^{1/108}\right)$

$= \sqrt[108]{2}$

⑦ $\left[\frac{8^{1/4} \times 32^{1/8}}{\sqrt[16]{1024}}\right]^9 = \left[\frac{(2^3)^{1/4} \times (2^5)^{1/8}}{(2^{10})^{1/16}}\right]^9$

$= \left(\frac{2^{3/4} \times 2^{5/8}}{2^{10/16}}\right)^9 = \left[(2)^{\frac{3}{4}+\frac{5}{8}-\frac{10}{16}}\right]^9$

$= \left[(2)^{\frac{3}{4}+\frac{5}{8}-\frac{5}{8}}\right]^9 = \left(2^{3/4}\right)^9 = (2)^{27/4}$

⑧ $\sqrt[16]{\left[\frac{\sqrt[7]{3} \times \sqrt[14]{81} \times (27)^{-9}}{243^{8/7}}\right]^{42}}$

$= \left[\left(\frac{(3)^{1/7} \times (3^4)^{1/14} \times (3^3)^{-9}}{(3^5)^{8/7}}\right)^{42}\right]^{1/16} = \left(\frac{3^{1/7} \times 3^{2/7} \times 3^{-189/7}}{3^{40/7}}\right)^{42/16}$

$= \left[(3)^{\frac{1}{7}+\frac{2}{7}-\frac{189}{7}-\frac{40}{7}}\right]^{21/8} = (3)^{\frac{-226}{7}\times\frac{21}{8}} = (3)^{\frac{-113\times3}{4}}$

$= (3)^{-339/4}$

⑨ $\left[\left(\sqrt[3]{4}\right)^{8}\right]^{11/2} \times \left[\left(\sqrt[9]{32}\right)^{-8}\right]^{17/2}$

$= \left\{\left[\left(2^{2}\right)^{1/3}\right]^{8}\right\}^{11/2} \times \left\{\left[\left(2^{5}\right)^{1/9}\right]^{-8}\right\}^{17/2}$

$= 2^{2\times\frac{1}{3}\times 8\times\frac{11}{2}} \times 2^{5\times\frac{1}{9}\times -8\times\frac{17}{2}}$

$= 2^{\frac{176}{6}} \times 2^{\frac{-680}{18}} = (2)^{\frac{528}{18}-\frac{680}{18}}$

$= (2)^{-152/18} = (2)^{-76/9} = \frac{1}{(2^{76/9})}$

$= \frac{1}{\sqrt[9]{2^{76}}}$

⑩ $\text{Log}_{\sqrt[5]{64}} \sqrt[8]{512} = \frac{\text{Log}\sqrt[8]{512}}{\text{Log}\sqrt[5]{64}}$

$= \frac{\text{Log}\,(2^{9})^{1/8}}{\text{Log}\,(2^{6})^{1/5}} = \frac{\text{Log}\,(2)^{9/8}}{\text{Log}\,(2)^{6/5}}$

$= \frac{9/8\ \text{Log}2}{6/5\ \text{Log}2} = \frac{9}{8}\times\frac{5}{6} = \frac{45}{48} = \left(\frac{15}{16}\right)$

⑪ $\text{Log}_{\sqrt[8]{81}} \sqrt[18]{729} = \frac{\text{Log}\sqrt[18]{729}}{\text{Log}\sqrt[8]{81}} = \frac{\text{Log}\,(3^{6})^{1/18}}{\text{Log}\,(3^{4})^{1/8}}$

$= \frac{\text{Log}\,(3)^{1/3}}{\text{Log}\,(3)^{1/2}} = \frac{\frac{1}{3}\text{Log}3}{\frac{1}{2}\text{Log}3} = 2/3$

(12) $\left(\frac{\text{Log}\sqrt[3]{16} \times \text{Log}\sqrt[9]{2187}}{\text{Log}(\sqrt{3})^{19} \times \text{Log}\sqrt[15]{1024}}\right)$

$$= \frac{\text{Log}(2^4)^{1/3} \times \text{Log}(3^7)^{1/9}}{\text{Log}(3^{1/2})^{19} \times \text{Log}(2^{10})^{1/15}} = \frac{\frac{4}{3}\text{Log}2 \times \frac{7}{9}\text{Log}3}{\frac{19}{2}\text{Log}3 \times \frac{2}{3}\text{Log}2}$$

$$= \frac{4}{3} \times \frac{7}{9} \times \frac{2}{19} \times \frac{3}{2} = \left(\frac{28}{171}\right)$$

(13) If $\text{Log}2 = x$, $\text{Log}3 = y$ Find $\text{Log}(1.20)$

(a) $2x+3y-1$ (b) $2x+y-1$ (c) $x+y+3$ (d) None of these

$$\text{Log}(1.20) = \text{Log}\left(\frac{12}{10}\right) = \text{Log}12 - \text{Log}10$$
$$= \text{Log}(2\times2\times3) - 1$$
$$= \text{Log}2 + \text{Log}2 + \text{Log}3 - 1$$
$$= x + x + y - 1$$
$$= 2x + y - 1$$

(14) If $\text{Log}2 = x$, $\text{Log}3 = y$ Find $\text{Log}(7.20)$

$$\text{Log}(7.20) = \text{Log}\left(\frac{72}{10}\right) = \text{Log}\left(\frac{2\times3\times2\times3\times2}{10}\right)$$

$$= \text{Log}2 + \text{Log}3 + \text{Log}2 + \text{Log}3 + \text{Log}2 - \text{Log}10$$
$$= x + y + x + y + x - 1$$
$$= 3x + 2y - 1$$

(15) $\text{Log}_{(3/2)}x = -5$. Find value of x.

$\Longrightarrow \quad \log_{(3/2)}(x) = -5$

If $\log_b a = k$ then $b^k = a$

$\therefore \quad \left(\frac{3}{2}\right)^{-5} = x$

$$x = \frac{3^{-5}}{2^{-5}} = \frac{2^5}{3^5} = \left(\frac{32}{243}\right)$$

(16) $\log_x 16 = \log_9 64$. Find value of x

$$\Longrightarrow \quad \frac{\log 16}{\log x} = \frac{\log 64}{\log 9}$$

$$\frac{4 \cdot \log 2}{\log x} = \frac{6 \log 2}{2 \log 3}$$

$$\frac{\log x}{4} = \frac{\log 3}{3}$$

$$\log x = \frac{4}{3} \log 3$$

$$\log x = \log (3)^{4/3}$$

$$x = (3)^{4/3} = \sqrt[3]{3^4} = \sqrt[3]{81}$$

(17) $\log_2 \log_2 \log_2 16 = ?$

$= \log_2 \log_2 \left(\frac{\log 16}{\log 2}\right)$

$= \log_2 \log_2 \left(\frac{4 \cdot \log 2}{\log 2}\right)$

$= \log_2 (\log_2 4)$

$= \log_2 \left(\frac{\log 4}{\log 2}\right)$

$= \log_2 \left(\frac{2 \log 2}{\log 2}\right)$

$= \log_2 2$

$= 1$

(18) $\log_x 9 = 8$. Find value of $x\sqrt{x}$

$\Rightarrow \log_x 9 = 8$

$\therefore x^8 = 9$

$(x^8)^{1/8} = (9)^{1/8}$

$x^1 = (3^2)^{1/8}$

$x = 3^{1/4}$

$x\sqrt{x}$

$= 3^{1/4} \times \sqrt{3^{1/4}}$

$= 3^{1/4} \times (3^{1/4})^{1/2}$

$= 3^{1/4} \times 3^{1/8}$

$= 3^{\frac{1}{4}+\frac{1}{8}}$

$= 3^{12/32}$

$= 3^{3/8} = \sqrt[8]{3^3}$

$= \sqrt[8]{27}$

(19) $\log_x 27 = 81$ Find value of $\sqrt[3]{x} \times \sqrt[5]{x}$

$\Rightarrow \log_x 27 = 81$

$x^{81} = 27$

$x = (27)^{1/81}$

$x = (3^3)^{1/81}$

$x = 3^{1/27}$

$\sqrt[3]{x} \times \sqrt[5]{x}$

$= x^{\frac{1}{3}} \times x^{\frac{1}{5}}$

$= (x)^{\frac{1}{3}+\frac{1}{5}} = x^{\frac{8}{15}}$

$= \left[(3)^{1/27}\right]^{8/15}$

$= (3)^{8/405}$

(20) $\log_3 x = 4$ Find value of $(\sqrt{x})^x$

$\Rightarrow \log_3 x = 4$

$\therefore 3^4 = x$

$x = 81$

$(\sqrt{x})^x$

$= (\sqrt{81})^{81}$

$= 9^{81} = (3^2)^{81}$

$= 3^{162}$

(21) $\log_{\sqrt[7]{2}} (32)^{-5} = m$. Find m.

$\Rightarrow m = \dfrac{\log 32^{-5}}{\log \sqrt[7]{2}} = \left(\dfrac{\log (2^5)^{-5}}{\log (2)^{1/7}}\right)$

$m = \left(\dfrac{\log 2^{-25}}{\log 2^{1/7}}\right) = \left(\dfrac{-25 \log 2}{\frac{1}{7} \log 2}\right)$

$m = -175$

(22) $\log_{\sqrt[9]{243}} \left(\sqrt[8]{81}\right)^{-1/8} = ?$

$\Rightarrow \dfrac{\log\left(\sqrt[8]{81}\right)^{-1/8}}{\log \sqrt[9]{243}} = \dfrac{\log\left[(3^4)^{1/8}\right]^{-1/8}}{\log (3^5)^{1/9}}$

$= \dfrac{\log 3^{-1/16}}{\log 3^{5/9}} = \dfrac{-\frac{1}{16} \log 3}{\frac{5}{9} \log 3} = -\frac{1}{16} \times \frac{9}{5}$

$= (-9/80)$

(23) $5\log x = \frac{1}{2}\log \sqrt[5]{30}$. Find value of x

$$\Rightarrow 5\log x = \frac{1}{2}\log (30)^{1/5}$$

$$\log x^5 = \log\left[(30)^{1/5}\right]^{1/2}$$

$$\log x^5 = \log 30^{1/10}$$

$$x^5 = 30^{1/10}$$

$$(x^5)^{1/5} = \left(30^{1/10}\right)^{1/5}$$

$$x = 30^{1/50} = \sqrt[50]{30}$$

(24) $$\left(\frac{\log_5 16 \times \log_8 125 \times \log_7 243}{\log_{\sqrt{7}} 729}\right)$$

$$= \left(\frac{\frac{\log 16}{\log 5} \times \frac{\log 125}{\log 8} \times \frac{\log 243}{\log 7}}{\frac{\log 729}{\log 7^{1/2}}}\right)$$

$$= \frac{4\log 2}{\log 5} \times \frac{3\log 5}{3\log 2} \times \frac{5\log 3}{\log 7} \times \frac{\frac{1}{2}\log 7}{6\log 3}$$

$$= \frac{4 \times 5 \times \frac{1}{2}}{6} = \frac{10}{6} = \left(\frac{5}{3}\right)$$

(25) $\log_{\sqrt{5}} x = 3$. Find value of $\left(x \times \sqrt[5]{x}\right)$

$\Rightarrow \log_{\sqrt{5}} x = 3$

$\therefore (\sqrt{5})^3 = x$

$x = (5^{1/2})^3$

$x = (5^{3/2})$

$x \times \sqrt[5]{x}$

$= 5^{3/2} \times (5^{3/2})^{1/5}$

$= 5^{3/2} \times 5^{3/10}$

$= (5)^{\frac{3}{2}+\frac{3}{10}} = (5)^{\frac{36}{20}}$

$= (5)^{9/5} = \sqrt[5]{5^9}$

(26) If $a^2+b^2 = 50$ & $ab = 20$ Find value of $(a+b)^2$

$\Rightarrow (a+b)^2 = a^2+b^2 + 2ab$

$= 50 + 2(20) = 90$

(27) $\log_2 \log_2 \log_2 \log_2 x = 0$ Find x

$\Rightarrow$

$\log_2 \left(\log_2 \log_2 \log_2 x\right) = 0$

$\log_2 \left(\log_2 \log_2 x\right) = 2^0 = 1$

$\log_2 \left(\log_2 x\right) = 2^1 = 2$

$\log_2 x = 2^2 = 4$

$2^4 = x$

$\therefore x = 16$

If $\log_b a = k$ then $b^k = a$

cross-check

$= \log_2 \log_2 \log_2 (\log_2 16)$

$= \log_2 \log_2 (\log_2 4)$

$= \log_2(\log_2 2)$

$= \log_2 1$

$= 0$

(28) $\log_a x = a$ then $x = ?$

$\Rightarrow \log_a x = a$ then

$a^a = x$

$\therefore x = a^a$

(29) $\log(1+2+3) = \log 1 + \log 2 + \log 3$

(a) True (b) False

$\Rightarrow \log(1+2+3)$

$= \log 6$

$= \log(1 \times 2 \times 3)$

$= \log 1 + \log 2 + \log 3$

(30) $\log(5+6+7) = \log 5 + \log 6 + \log 7$

(a) True (b) False

L.H.S. $= \log(5+6+7) = \log 18$

R.H.S. $= \log 5 + \log 6 + \log 7 = \log(5 \times 6 \times 7) = \log 210$

(31) $\log_2 x + \log_8 x + \log_{32} x = 23/5$

Find value of x.

$\Rightarrow \frac{\log x}{\log 2} + \frac{\log x}{\log 8} + \frac{\log x}{\log 32} = \frac{23}{5}$

$\left(\frac{\log x}{\log 2}\right) + \left(\frac{\log x}{3\log 2}\right) + \left(\frac{\log x}{5 \cdot \log 2}\right) = \frac{23}{5}$

$\frac{15 \cdot \log x}{15 \cdot \log 2} + \frac{5 \log x}{15 \log 2} + \frac{3 \log x}{15 \log 2} = \frac{23}{5}$

$\frac{23 \log x}{15 \log 2} = \frac{23}{5}$

$$\frac{\log x}{\log 2} = \frac{15}{5} = 3$$

$$\log x = 3\log 2$$

$$\log x = \log 8$$

$$\boxed{x = 8}$$

(32) $\log 5 = 0.6990$

$\log 3 = 0.4771$

Find value of $\log\left(\frac{50}{3,00,000}\right)$

$\Rightarrow \log(50/300,000)$

$= \log 50 - \log 3,00,000$

$= 1.6990 - 5.4771 = -3.7781$

(33) $\log 2 = 0.3010$, $\log 3 = 0.4771$

Find $\log\left(\frac{0.003}{2000}\right)$

$\Rightarrow$

$= \log 0.003 - \log 2000$

$= (\bar{3}.4771) - (3.3010)$

$= (-3 + 0.4771) - 3.3010$

$= -2.5229 - 3.3010$

$= -5.8239$

OR

$\log\left(\frac{0.003}{2000}\right)$

$= \log\left(\frac{3}{2000000}\right)$

$= \log 3 - \log 20,00,000$

$= 0.4771 - 6.3010$

$= -5.8239$

(34) If $2s : 3t$ is duplicate ratio of $(2s-p) : (3t-p)$ then

~~(a)~~ $p^2 = 6st$ (b) $p = 6st$ (c) $2p = 3st$ (d) None of these

$\Rightarrow$

$$\frac{2s}{3t} = \frac{(2s-p)^2}{(3t-p)^2}$$

$$\frac{2s}{3t} = \frac{4s^2 - 4sp + p^2}{9t^2 - 6tp + p^2}$$

$$2s(9t^2 - 6tp + p^2) = 3t(4s^2 - 4sp + p^2)$$

$$18t^2s - 12tps + 2p^2s = 12ts^2 - 12tps + 3tp^2$$

$$18t^2s - 12ts^2 = 3tp^2 - 2p^2s$$

$$6ts(3t - 2s) = p^2(3t - 2s)$$

$$\therefore p^2 = 6st$$

(35) $\frac{A}{3} = \frac{B}{8} = \frac{C}{7}$ Find $A:B:C = ?$

(a) 7:8:3 (b) 3:8:7 (c) 8:7:3 (d) None of these

$$\frac{A}{3} = \frac{B}{8} \quad \therefore \frac{A}{B} = \frac{3}{8} \qquad A:B = 3:8$$

$$\frac{B}{8} = \frac{C}{7} \qquad \frac{B}{C} = \frac{8}{7} \qquad B:C = 8:7$$

$$\therefore A:B:C = 3:8:7$$

(36) Anand earns ₹5000 in 6 hrs & Vinod earns ₹7000 in 8 hrs. Find ratio of their earning per hour.

$$\Rightarrow \frac{5000/6}{7000/8} = \frac{5000}{6} \times \frac{8}{7000} = \frac{4}{3} = 20:21$$

(37) Monthly Income of A, B is in the ratio of 5:6 & Monthly expenses are in the ratio of 9:11. If both of them save ₹50 p.m. Find their Incomes?

$\Rightarrow$ monthly incomes are 5x, 6x resp.

$$\frac{5x-50}{6x-50} = \frac{9}{11}$$

$$55x - 550 = 54x - 450$$

$$x = 100$$

$\therefore$ Monthly incomes of A, B are ₹ 500, ₹600

(38) Duplicate ratio of sub-duplicate ratio of 8:9 is :

(a) 64:81 (b) 9:8 (c) 128 : 162 ~~(d)~~ None of these

(39) What least number should be added to each one of 6, 14, 18, 38 to make them in proportion ?

(a) 5 (b) 3 ~~(c)~~ 2 (d) 4

(6+2), (14+2), (18+2), (38+2) i.e. 8, 16, 20, 40 are in proportion

(40) Raja can walk certain distance in 50 days if he takes rest for 9 hrs per day. How long it will take him to walk twice as far if he walks twice as fast and rests twice as long each day ?

~~(a)~~ 125 days (b) 50 days (c) 100 days (d) None

	original	New
walking speed	x kms/hr	$2x$ kms/hr
Rest per day	9 hrs	18 hrs
walks per day	15 hrs	6 hrs
Distance per day	$15x$ kms	$2x \times 6$ hrs $= 12x$ kms
No. of days	50 days	$\frac{1500x}{12x} = 125$ days
Total Distance Traveled	$15x \times 50$ days $= 750x$ kms	$1500x$

(41) $3, x, 27, y$ are in continued proportion then $x : y = ?$

(a) 9:1 (b) 1:9 (c) 1:27 (d) 3:1

$\Rightarrow \frac{3}{x} = \frac{x}{27} = \frac{27}{y}$

$x^2 = 81 \quad \therefore x = 9$

$3, 9, 27, 81$ are in continued prop^n

$\therefore y = 81$

$x : y = 9 : 81 = 1 : 9$

(42) $\log_{1/9} 243 = x$. Find x.

$\Rightarrow \log_{1/9} 243 = x$

$\therefore \left(\frac{1}{9}\right)^x = 243$ ------

$\left(\frac{1}{3^2}\right)^x = 3^5$

$\left(3^{-2}\right)^x = (3)^5$

If $\log_b a = k$ then $b^k = a$

$3^{-2x} = 3^5$

$\therefore -2x = 5 \quad \therefore x = -5/2$

(43) $\log_2 x + \log_4 x + \log_{16} x = 21/4$ then $x = ?$

(a) 7 (b) 8 (c) 9 (d) 10

$\Rightarrow \frac{\log x}{\log 2} + \frac{\log x}{2\log 2} + \frac{\log x}{4\log 2} = \frac{21}{4}$

$\frac{4\log x}{4\log 2} + \frac{2\log x}{4\log 2} + \frac{\log x}{4.\log 2} = \frac{21}{4}$

$\frac{7\log x}{4.\log 2} = \frac{21}{4}$

$\frac{\log x}{\log 2} = \frac{21}{4} \times \frac{4}{7} = 3$

$\log_2 x = 3$

$\therefore 2^3 = x$

$\therefore x = 8$

(44) $\log_a 3 = 2$, $\log_b 8 = 3$ then $\log_6 a = ?$

(a) $\log_3 2$ (b) $\log_2 3$ (c) $\log_3 4$ (d) $\log_4 3$

$\Rightarrow \quad \log_a 3 = 2$

$\therefore \quad a^2 = 3$

$a = 3^{1/2}$

$\log_b 8 = 3$

$\therefore \quad b^3 = 8$

$b = 2$

$\log_b a = \frac{\log a}{\log b} = \frac{\log 3^{1/2}}{\log 2}$

$= \frac{\frac{1}{2}\log 3}{\log 2} = \frac{\log 3}{2\log 2}$

$= \frac{\log 3}{\log 2^2} = \frac{\log 3}{\log 4}$

$= \log_4 3$

(45) Find x If $\log_x 10 + \log_x 100 + \log_x 1000 = 6$

~~(a)~~ 10 (b) 2 (c) 4 (d) 6

$\log_{10} 10 = 1$ $\quad \therefore x = 10$

$\log_{10} 100 = 2$

$\log_{10} 1000 = 3$

(46) $2\log a + 3\log b - 2 = 0$ then $a^2 b^3 = ?$

(a) 10^4 (b) 10^3 ~~(c)~~ 10^2 (d) None of these

$\Rightarrow \quad 2\log a + 3\log b - 2 = 0$

$\log a^2 + \log b^3 = 2$

$\log_{10}(a^2 b^3) = 2 \quad \therefore 10^2 = a^2 b^3$

$\therefore (a^2 b^3) = 10^2$

(47) $\log_2\left[\log_2\left\{\log_3\left(\log_3 27^3\right)\right\}\right] = ?$

(a) 1/2 (b) 1 ~~(c)~~ 0 (d) 2

$= \log_2\left[\log_2\left\{\log_3\left(\frac{9\log 3}{\log 3}\right)\right\}\right]$

$= \log_2\left[\log_2\{\log_3 9\}\right] = \log_2\left[\log_2(2)\right]$

$= \log_2(1) = \text{zero}$

(48) $2\log x = 4\log 4$ then $x = ?$

$\Rightarrow 2\log x = 4 \cdot \log 4$

$\log x = 2 \cdot \log 4$

$\log x = \log 4^2$

$\log x = \log 16$

$x = 16$

$\log x^2 = \log 4^4$

$\therefore x^2 = 256$

$x = 16$

(49) Age of a person is twice the sum of ages of his 2 sons & 5 years ago his age was 3 times the sum of ages of his 2 sons, his present age is ____ years

(a) 60 (b) 52 (c) 51 ~~(d) 50~~

Let sum of ages of his 2 sons $= x$ } present

Age of a person $= 2x$ }

$2x - 5 = 3(x - 10)$

$2x - 5 = 3x - 30$

$\boxed{25 = x}$

$\therefore$ his present age $= 2x = 2 \times 25 = 50$ years

(50) Sum of 2 numbers is 45 and the mean proportional between them is 18. The numbers are:

(a) 15,30 (b) 32,13 ~~(c) 36,9~~ (d) 25,20

$36 + 9 = 45$

$\sqrt{36 \times 9} = 18$

(51) If $(7p+3q) : (3p-2q) = 2:1$ then $p:q = ?$

$\Rightarrow \frac{7p+3q}{3p-2q} = \frac{2}{1}$

$7p+3q = 6p-4q$

$p = -7q$

$\therefore (p/q) = (-7/1)$

$= -7:1$

(52) If $\text{Log}_a 23 = b$ then

(a) $a^{23} = b$ (b) $b^q = 23$ (c) $b^q = 23$ ~~(d)~~ $a^b = 23$

(53) The angles of a triangle are in the ratio of $2:29:5$ then Find the angles:

$\Rightarrow 2x + 29x + 5x = 180^\circ$

$36x = 180$

$x = 5$

$\therefore$ Angles are $10^\circ, 145^\circ, 25^\circ$

(54) $a^2+b^2 = 45$, $ab = 18$ Find $\left(\frac{1}{a} + \frac{1}{b}\right) = ?$

$\Rightarrow (a+b)^2 = a^2+b^2+2ab$

$(a+b)^2 = 45 + 2(18) = 81$

$\therefore (a+b) = 9$

$\frac{1}{a}+\frac{1}{b}$

$= \left(\frac{a+b}{ab}\right) = \left(\frac{9}{18}\right) = 1/2$

(55) If $a^2 . \text{Log}_3 x = b . \text{Log}_{27} x$ then

(a) $a = 3$ ~~(b)~~ $3a^2 = b$ (c) $b^2 = 3a$ (d) None of these

$\Rightarrow a^2 . \text{Log}_3 x = b . \text{Log}_{27} x$

$a^2 \times \frac{\text{Log}x}{\text{Log}3} = \frac{b . \text{Log}x}{3 . \text{Log}3}$

$a^2 = \frac{b}{3}$ $\therefore 3a^2 = b$

(56) Mr. A says to his son '7 years before I was 7 times as old as you were' and 3 years later I shall be 3 times as old as you will be. Find present age of Mr. A's son:

(a) 12 years (b) 15 years (c) 5 years (d) 7 years

present age of Mr. A = x

Mr A's son = y

$(x-7) = 7(y-7)$

$x-7 = 7y-49$

$x-7y = -42$ ---- (1)

$(x+3) = 3(y+3)$

$x+3 = 3y+9$

$x-3y = 6$ ---- (2)

$x - 7y = -42$

$-x + 3y = -6$

$-4y = -48$

$y = 12$ years

(57) one third of one half of three fourth of a number is 60, the number is:

(a) 480 (b) 520 (c) 500 (d) None of these

$\Rightarrow \frac{1}{3} \times \frac{1}{2} \times \frac{3}{4} \times x = 60$

$x = 480$

(58) $x^2 + y^2 = 14xy$ then $2\log 4 + \log x + \log y = ?$

(a) $\frac{1}{2}\log(x+y)$ (b) $2\log(x+y)$ (c) $\log(x+y)$ (d) None

$\Rightarrow$

$14xy = x^2+y^2$

$14xy + 2xy = x^2+y^2+2xy$

$16xy = (x+y)^2$

$\log(16xy) = \log(x+y)^2$

$= 2.\log(x+y)$

$2\log 4 + \log x + \log y$

$= \log 16 + \log x + \log y$

$= \log(16xy)$

$= 2.\log(x+y)$

(59) $(2p^2q^3r^2)^0 = ?$

(a) 0 (b) 1 (c) $4p^4q^6r^4$ (d) None

(60) $x^{1/p} = y^{1/q} = z^{1/r}$ and $xyz = 1$ then $(p+q+r) = ?$

(a) 1 (b) 0 (c) 1/2 (d) None

$\Rightarrow x^{1/p} = y^{1/q} = z^{1/r} = k$

$xyz = 1$

$k^p \times k^q \times k^r = k^0$

$\therefore x^{1/p} = k$

$(x^{1/p})^p = k^p$

$x = k^p$

$y = k^q$

$z = k^r$

$(k)^{p+q+r} = k^0$

$\therefore (p+q+r) = 0$

(61) $a^x = b$, $b^y = c$, $c^z = a$ then $xyz = ?$

(a) 1 (b) 2 (c) 3 (d) None

$a^x = b$

$\log a^x = \log b$

$x \cdot \log a = \log b$

$x = \frac{\log b}{\log a}$

$y = \frac{\log c}{\log b}$

$z = \frac{\log a}{\log c}$

$xyz = \frac{\log b}{\log a} \times \frac{\log c}{\log b} \times \frac{\log a}{\log c} = 1$

OR

$b^y = c$, $c^z = a$

$(b^y)^{1/y} = c^{1/y}$

$b = c^{1/y}$

$c = a^{1/z}$

$a^x = b$

$a^x = c^{1/y}$

$a^x = (a^{1/z})^{1/y}$

$a^x = a^{\frac{1}{yz}}$

$\therefore x = \frac{1}{yz}$

$xyz = 1$

(62) $\log_{2\sqrt{3}} 1728 = m$ then $m = ?$

(a) 2 (b) 3 (c) 4 (d) None

$\Rightarrow \log_{2\sqrt{3}} 1728 = m$

$\therefore (2\sqrt{3})^m = 1728 = 64 \times 27 = 2^6 \times 3^3 = 2^6 \times [(\sqrt{3})^2]^3$

$(2\sqrt{3})^m = 2^6 \times \sqrt{3}^6$

$(2\sqrt{3})^m = (2\sqrt{3})^6$

$\therefore m = 6$

(63) $\left(a^m / a^{-m}\right) = ?$ $\quad \left(\frac{a^m}{a^{-m}}\right) = a^m \times a^m = a^{m+m} = a^{2m}$

(a) 1 (b) −1 (c) 0 ~~(d)~~ None

(64) $a:b = 2:5$, $b:c = 15:46$, $c:d = 92:200$ then $a:d = ?$

(a) 2:3 (b) 3:192 (c) 1:100 ~~(d)~~ None of these

$\Rightarrow$

$$\frac{a}{d} = \left(\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d}\right) = \frac{2}{\cancel{5}} \times \frac{\overset{3}{\cancel{15}}}{\cancel{46}} \times \frac{\cancel{92}^{\,2}}{\cancel{200}_{\,50}} = 3:50$$

Compounded ratio of $a:b$, $b:c$, $c:d$ is $a:d$

(65) $\text{Log}_4(x^2+x) - \text{Log}_4(x+1) = 2$ then $x = ?$

~~(a)~~ 16 (b) 4 (c) 8 (d) None

$\Rightarrow$ $\text{Log}_4\left(\frac{x^2+x}{x+1}\right) = 2$

$$\frac{x^2+x}{x+1} = 4^2$$

$$\frac{x(x+1)}{(x+1)} = 16 \quad \therefore x = 16$$

(66) Characteristic of 7.128 is

(a) 6 (b) $\overline{8}$ ~~(c)~~ 0 (d) None

(67) $2^x - 2^{x-1} = 4$ then $x^x = ?$

(a) 2 (b) 1 (c) 64 ~~(d)~~ 27

$\Rightarrow$ $2^x - 2^{x-1} = 4$

$2^x - \frac{2^x}{2^1} = 4$

$\therefore x^x = 3^3 = 27$

$$2^x\left(1-\frac{1}{2}\right) = 4$$

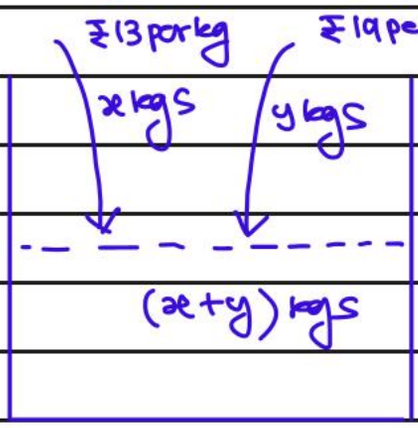
$$2^x \times \frac{1}{2} = 4$$

$\therefore \quad 2^x = 8 = 2^3$

$\therefore \quad x = 3$

(68) 2 kinds of rice, 1st costs ₹13 per kg & 2nd costs ₹19 per kg are mixed together. Find in what ratio they are mixed so that mixture costs ₹ 14.20 per kg?

(a) 3:1 ~~(b) 4:1~~ (c) 3:4 (d) 4:3



$$14.20(x+y) = (13x + 19y)$$

$$14.20x + 14.20y = 13x + 19y$$

$$1.20x = 4.80y$$

$$\frac{x}{y} = \frac{4.80}{1.20} = \frac{4}{1} = 4:1$$

(69) If $x = 3^{1/3} + 3^{-1/3}$ then $3x^3 - 9x = ?$

(a) 15 ~~(b) 10~~ (c) 12 (d) None

$\Longrightarrow$

$x = 3^{1/3} + 3^{-1/3}$ $\qquad (a+b)^3 = a^3+b^3+3ab(a+b)$

Taking cube on both sides

$$x^3 = \left(3^{1/3} + 3^{-1/3}\right)^3$$

$$x^3 = \left(3^{1/3}\right)^3 + \left(3^{-1/3}\right)^3 + 3 \times 3^{1/3} \times 3^{-1/3}\left(3^{1/3} + 3^{-1/3}\right)$$

$$x^3 = 3^1 + 3^{-1} + 3 \times 3^{\frac{1}{3}-\frac{1}{3}} \times x$$

$$x^3 = 3 + \frac{1}{3} + 3x$$

$$3x^3 = 9 + 1 + 9x$$

$$3x^3 - 9x = 10$$

① **Commensurable ratio** : If ratio of 2 or more terms **can be** written in the form of

Ratio of Integers

then it is said to be a commensurable ratio

② **Incommensurable ratio OR Non-commensurable ratio**

If ratio of 2 or more terms **can not** be written in the form of

Ratio of Integers

then it is said to be a incommensurable ratio OR Non commensurable ratio

Ratio	commensurable / incommensurable ratio
5 : 18	It is a ratio of integers ∴ commensurable ratio
8.50 : 3.50	It can written in form of ratio of integers i.e. 17 : 7 = 85 : 35 ∴ It is a commensurable ratio
8.15 : 2.221	It can be written as 8150 : 2221 ∴ commensurable ratio
5.8361 : 3	It can be written as 58,361 : 30,000 ∴ commensurable ratio
$\sqrt{5} : \sqrt{7}$	It is incommensurable ratio as it can not be written in the form of ratio of integers

$\sqrt{25} : \sqrt{64}$	It can be written as 5:8 $\therefore$ commensurable ratio
$\sqrt{50} : \sqrt{450}$	$\frac{\sqrt{50}}{\sqrt{450}} = \frac{\sqrt{50}}{\sqrt{9 \times 50}} = \frac{\sqrt{50}}{\sqrt{9} \times \sqrt{50}} = \frac{1}{3}$ $= 1:3$ $\therefore$ It is commensurable ratio
$\sqrt{147} : \sqrt{75}$	$\frac{\sqrt{147}}{\sqrt{75}} = \frac{\sqrt{49 \times 3}}{\sqrt{25 \times 3}} = \frac{\sqrt{49} \times \sqrt{3}}{\sqrt{25} \times \sqrt{3}} = 7:5$ It is commensurable ratio
$8\frac{1}{3} : 9\frac{2}{3}$	$\frac{25}{3} : \frac{29}{3} = 25:29$ $\therefore$ It is commensurable ratio
$8\frac{1}{7} : 11\frac{2}{3}$	$\frac{57}{7} : \frac{35}{3} = 171:245$ $\therefore$ It is commensurable ratio
$\sqrt{5} : \sqrt{10}$	It can not be expressed in the form of ratio of integers $\therefore$ It is incommensurable ratio
$\sqrt{100} : \sqrt{196}$	$10:14 = 5:7$ $\therefore$ It is commensurable ratio

(70) 2 whole numbers whose sum is 100 can not be in the ratio of ____

(a) 16:9 ~~(b)~~ 3:4 (c) 3:7 (d) 4:1

$\Rightarrow$ 16:9 $\Rightarrow$ 64, 36

3:7 $\Rightarrow$ 30, 70

4:1 $\Rightarrow$ 80, 20

(71) Divide ₹61,000 among a, b, c such that a & b get in the ratio of 2:5 and b & c get in the ratio of 3:8

$\Rightarrow$ $a:b = 2:5 = 6:15$

$b:c = 3:8 = 15:40$

$\therefore\ a:b:c = 6:15:40$

Share of

$a = \frac{6}{61} \times 61{,}000 = 6{,}000$

$b = 15{,}000$

$c = 40{,}000$

(72) $\left(\frac{2^{1/3} \times 2^{1/5}}{4^{2/3}}\right) = \frac{2^{\frac{1}{3}+\frac{1}{5}}}{(2^2)^{2/3}} = \frac{2^{\frac{8}{15}}}{2^{4/3}} = 2^{\frac{8}{15}-\frac{4}{3}} = (2)^{\frac{8}{15}-\frac{20}{15}}$

$= (2)^{-12/15} = (2)^{-4/5} = \left[\frac{1}{(2)^{4/5}}\right]$

(73) A & B together can finish the work in 8 days. If A alone can finish the work in 24 days then B alone can finish the work in ____ no. of days.

(a) 10 ~~(b)~~ 12 (c) 15 (d) None

$\Rightarrow$ 'B' alone can complete the work in 'x' days

$\left(\begin{matrix}\text{work of A in}\\ \text{one day}\end{matrix}\right) + \left(\begin{matrix}\text{work of B}\\ \text{in one day}\end{matrix}\right) = \left(\begin{matrix}\text{work of A \& B}\\ \text{in one day}\end{matrix}\right)$

$\frac{1}{24} + \frac{1}{x} = \frac{1}{8}$

$\frac{x+24}{24x} = \frac{1}{8}$

$8x + 192 = 24x$

$192 = 16x$

$\therefore\ x = 12$ days

(74) $\log_{2\sqrt{3}} 1728 = m$. Find value of m^2 is

(a) 6 (b) 36 (c) $\sqrt{6}$ (d) None of these

$\Rightarrow \log_{2\sqrt{3}} 1728 = m$

$\therefore (2\sqrt{3})^m = 1728 = 2^6 \times 3^3 = 2^6 \times (\sqrt{3})^6 = (2\sqrt{3})^6$

$\therefore \boxed{m = 6}$ $\therefore m^2 = 6^2 = 36$

(75) $2\log x = 15 \cdot \log \sqrt[7]{2}$. Find value of x.

$\Rightarrow \log x^2 = \log (\sqrt[7]{2})^{15}$

$x^2 = (\sqrt[7]{2})^{15} = (2^{1/7})^{15} = 2^{15/7}$

$x^2 = 2^{15/7}$

$(x^2)^{1/2} = (2^{15/7})^{1/2}$ $\therefore x = 2^{15/14} = \sqrt[14]{2^{15}}$

(76) If $a^x = b$, $b^y = c$, $c^z = a$ then Find value of $(xyz)^3$.

(a) 1 (b) 0 (c) 8 (d) None of these

$\Rightarrow$ $a^x = b$	$b^y = c$	$c^z = a$
$a^x = c^{1/y}$	$(b^y)^{1/y} = c^{1/y}$	$(c^z)^{1/z} = a^{1/z}$
$a^x = (a^{1/z})^{1/y}$	$\therefore b = c^{1/y}$	$c = a^{1/z}$
$a^x = a^{1/zy}$		
$\therefore x = \frac{1}{zy}$		
$\therefore xyz = 1$ $\therefore (xyz)^3 = 1$		

(77) If $2^x - 2^{x-1} = 16$. Find value of $\sqrt{x}$.

$\Rightarrow 2^x - \left(\frac{2^x}{2^1}\right) = 16$

$\left(1 \times 2^x\right) - \left(2^x \times \frac{1}{2}\right) = 16$

$2^x\left(1-\frac{1}{2}\right) = 16$

$2^x \times 1/2 = 16$

$\therefore 2^x = 32 = 2^5 \quad \therefore x = 5$

$\therefore \sqrt{x} = \sqrt{5}$

(78) $\log(m+n) =$

(a) $\log m + \log n$ (b) $\log m \times \log n$

(c) $\log m / \log n$ (d) None of these

(79) If $\log 20 = 1.3010$. Find $\log 0.00000020$

$\Rightarrow \log 0.00000020 = \bar{7}.3010$

$= -7 + 0.3010 = -6.6990$

(80) $\left(a^{1/8} + a^{-1/8}\right)\left(a^{1/8} - a^{-1/8}\right)\left(a^{1/4} + a^{-1/4}\right)\left(a^{1/2} + a^{-1/2}\right) = ?$

(a) $a + 1/a$ (b) $a - 1/a$ (c) $a^2 + 1/a^2$ (d) None of these

$\Rightarrow \left(a^{1/8} + a^{-1/8}\right)\left(a^{1/8} - a^{-1/8}\right)\left(a^{1/4} + a^{-1/4}\right)\left(a^{1/2} + a^{-1/2}\right)$

$= \left[\left(a^{1/8}\right)^2 - \left(a^{-1/8}\right)^2\right]\left(a^{1/4} + a^{-1/4}\right)\left(a^{1/2} + a^{-1/2}\right)$

$= \left(a^{1/4} - a^{-1/4}\right)\left(a^{1/4} + a^{-1/4}\right)\left(a^{1/2} + a^{-1/2}\right)$

$= \left(a^{1/2} - a^{-1/2}\right)\left(a^{1/2} + a^{-1/2}\right) = a - a^{-1} = \left(a - \frac{1}{a}\right)$

(81) If $\log(2a-3b) = \log a - \log b$ then $a = ?$

(a) $3b^2/2b-1$ (b) $3b/2b-1$ (c) $b^2/2b+1$ (d) $3b^2/2b+1$

$\Rightarrow \log(2a-3b) = \log a - \log b$

$\log(2a-3b) = \log\left(\frac{a}{b}\right)$

$\therefore 2a-3b = \frac{a}{b}$

$2ab - 3b^2 = a$

$2ab - a = 3b^2$

$a(2b-1) = 3b^2$

$\therefore a = \left(3b^2/2b-1\right)$

(82) $\left(\frac{1}{\log_{ab} abc}\right) + \left(\frac{1}{\log_{bc} abc}\right) + \left(\frac{1}{\log_{ac} abc}\right) = ?$

(a) 0 (b) 1 ~~(c) 2~~ (d) −1

$$\Rightarrow \frac{1}{\frac{\log abc}{\log ab}} + \frac{1}{\frac{\log abc}{\log bc}} + \frac{1}{\frac{\log abc}{\log ac}}$$

$$= \frac{\log ab}{\log abc} + \frac{\log bc}{\log abc} + \frac{\log ac}{\log abc} = \frac{\log ab + \log bc + \log ac}{\log abc}$$

$$= \frac{\log(ab \times bc \times ac)}{\log abc} = \frac{\log(a^2b^2c^2)}{\log(abc)} = \frac{\log(abc)^2}{\log(abc)} = \frac{2.\log(abc)}{\log(abc)}$$

$$= 2$$

(83) ₹407 are to divided in the ratio of $\frac{1}{4} : \frac{1}{5} : \frac{1}{6}$ then smallest share is :

$$\Rightarrow \frac{1}{4} : \frac{1}{5} : \frac{1}{6} = 15 : 12 : 10$$

Smallest share $\Rightarrow \frac{10}{37} \times ₹407 = ₹110$

(84) Third proportional to (a^2-b^2) and $(a+b)^2$ is

(a) $\left(\frac{a+b}{a-b}\right)$ (b) $\left(\frac{a-b}{a+b}\right)$ (c) $\frac{(a-b)^2}{a+b}$ ~~(d)~~ $\frac{(a+b)^3}{(a-b)}$

$\Rightarrow$ Let 3rd proportional be 'm'

$\therefore (a^2-b^2), (a+b)^2, m$ are in proportion.

$$\left[(a+b)^2\right]^2 = (a^2-b^2) \times m$$

$$m = \frac{(a+b)^4}{(a^2-b^2)} = \frac{(a+b)(a+b)^3}{(a+b)(a-b)} = \frac{(a+b)^3}{(a-b)}$$

(85) If $x = y^a$, $y = z^b$, $z = x^c$ then $abc = ?$

(a) 2 (b) 1 (c) 3 (d) 4

$\Rightarrow$ $x = y^a$ | $y = z^b$ | $z = x^c$ | abc

$\log x = a \cdot \log y$ | $\log y = b \cdot \log z$ | $\log z = c \cdot \log x$ | $= \frac{\log x}{\log y} \times \frac{\log y}{\log z} \times \frac{\log z}{\log x}$

$a = \frac{\log x}{\log y}$ | $b = \frac{\log y}{\log z}$ | $c = \frac{\log z}{\log x}$ | $= 1$

(86) $\left(\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}\right) = ?$

(a) 1/2 (b) 3/2 (c) 2/3 (d) 1/3

$$= \left(\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}\right) = \left[\frac{\left(2^n + \frac{2^n}{2^1}\right)}{\left(2^n \times 2^1 - 2^n\right)}\right] = \left[\frac{2^n\left(1 + \frac{1}{2}\right)}{2^n(2-1)}\right] = \frac{3/2}{1}$$

$$= 3/2$$

(87) Find x If $x \cdot x^{1/3} = \left(x^{1/3}\right)^x$

(a) 3 (b) 4 (c) 2 (d) 6

$\Rightarrow$ $x^1 \cdot x^{1/3} = \left(x^{1/3}\right)^x$

$$(x)^{1+\frac{1}{3}} = (x)^{x/3}$$

$$(x)^{4/3} = (x)^{x/3}$$

$\therefore \frac{x}{3} = \frac{4}{3}$ $\therefore x = 4$

(88) $25^{150} = (25x)^{50}$ then $x = ?$

(a) 5^3 (b) 5^2 ~~(c)~~ 5^4 (d) 5^1

$(25)^{150} = (25x)^{50}$ $\quad \therefore x = \frac{25^3}{25^1} = 25^{3-1}$

$(25^3)^{50} = (25x)^{50}$ $\quad x = 25^2 = (5^2)^2 = 5^4$

$\therefore 25^3 = 25x$

(89) salary of p is 25% lower than q and salary of R 20% higher than q then ratio of salary of R and p will be

(a) 5:8 ~~(b)~~ 8:5 (c) 5:3 (d) 3:5

$\Rightarrow$ suppose q's salary = 100

$\therefore$ p's salary = 75

R's salary = 120

Ratio of salary of R & p = 120:75

= 8:5

(90) If $\log 2 = x$, $\log 3 = y$ then $\log 60 = ?$

~~(a)~~ $x+y+1$ (b) $x+y-1$ (c) $2x+3y+1$ (d) $2x+5y-1$

$\Rightarrow$ $\log(60) = \log(3 \times 2 \times 10)$

$= \log 3 + \log 2 + \log 10$

$= y + x + 1$

$= x+y+1$